

ADVANCED SUBSIDIARY GCE
MATHEMATICS
Further Pure Mathematics 1

4725

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

- 1 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & 2 \\ 3 & 4 \end{pmatrix}$ and $\mathbf{I}$ is the 2×2 identity matrix.
- (i) Find $\mathbf{A} - 4\mathbf{I}$. [2]
- (ii) Given that $\mathbf{A}$ is singular, find the value of a . [3]
- 2 The cubic equation $2x^3 + 3x - 3 = 0$ has roots α , β and γ .
- (i) Use the substitution $x = u - 1$ to find a cubic equation in u with integer coefficients. [3]
- (ii) Hence find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$. [2]
- 3 The complex number z satisfies the equation $z + 2iz^* = 12 + 9i$. Find z , giving your answer in the form $x + iy$. [5]
- 4 Find $\sum_{r=1}^n r(r+1)(r-2)$, expressing your answer in a fully factorised form. [6]
- 5 (i) The transformation T is represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Give a geometrical description of T . [2]
- (ii) The transformation T is equivalent to a reflection in the line $y = -x$ followed by another transformation S . Give a geometrical description of S and find the matrix that represents S . [4]
- 6 One root of the cubic equation $x^3 + px^2 + 6x + q = 0$, where p and q are real, is the complex number $5 - i$.
- (i) Find the real root of the cubic equation. [3]
- (ii) Find the values of p and q . [4]
- 7 (i) Show that $\frac{1}{r^2} - \frac{1}{(r+1)^2} \equiv \frac{2r+1}{r^2(r+1)^2}$. [1]
- (ii) Hence find an expression, in terms of n , for $\sum_{r=1}^n \frac{2r+1}{r^2(r+1)^2}$. [4]
- (iii) Find $\sum_{r=2}^{\infty} \frac{2r+1}{r^2(r+1)^2}$. [2]
- 8 The complex number a is such that $a^2 = 5 - 12i$.
- (i) Use an algebraic method to find the two possible values of a . [5]
- (ii) Sketch on a single Argand diagram the two possible loci given by $|z - a| = |a|$. [4]

9 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & 1 \\ 1 & 1 & a \end{pmatrix}$, where $a \neq 1$.

(i) Find $\mathbf{A}^{-1}$. [7]

(ii) Hence, or otherwise, solve the equations

$$2x - y + z = 1,$$

$$3y + z = 2,$$

$$x + y + az = 2. \quad [4]$$

10 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(i) Find $\mathbf{M}^2$ and $\mathbf{M}^3$. [3]

(ii) Hence suggest a suitable form for the matrix $\mathbf{M}^n$. [1]

(iii) Use induction to prove that your answer to part (ii) is correct. [4]

(iv) Describe fully the single geometrical transformation represented by $\mathbf{M}^{10}$. [3]

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