ADVANCED SUBSIDIARY GCE

MATHEMATICS

4725

Further Pure Mathematics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes

4725

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & 2 \\ 3 & 4 \end{pmatrix}$ and $\mathbf{I}$ is the 2×2 identity matrix.

(i) Find $\mathbf{A} - 4\mathbf{I}$. **[2]**

(ii) Given that $\mathbf{A}$ is singular, find the value of a. **[3]**

2 The cubic equation $2x^3 + 3x - 3 = 0$ has roots α, β and γ.

(i) Use the substitution $x = u - 1$ to find a cubic equation in u with integer coefficients. **[3]**

(ii) Hence find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$. **[2]**

3 The complex number z satisfies the equation $z + 2\mathrm{i}z^* = 12 + 9\mathrm{i}$. Find z, giving your answer in the form $x + \mathrm{i}y$. **[5]**

4 Find $\displaystyle\sum_{r=1}^{n} r(r + 1)(r - 2)$, expressing your answer in a fully factorised form. **[6]**

5 **(i)** The transformation T is represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Give a geometrical description of T. **[2]**

(ii) The transformation T is equivalent to a reflection in the line $y = -x$ followed by another transformation S. Give a geometrical description of S and find the matrix that represents S. **[4]**

6 One root of the cubic equation $x^3 + px^2 + 6x + q = 0$, where p and q are real, is the complex number $5 - \mathrm{i}$.

(i) Find the real root of the cubic equation. **[3]**

(ii) Find the values of p and q. **[4]**

7 **(i)** Show that $\dfrac{1}{r^2} - \dfrac{1}{(r + 1)^2} \equiv \dfrac{2r + 1}{r^2(r + 1)^2}$. **[1]**

(ii) Hence find an expression, in terms of n, for $\displaystyle\sum_{r=1}^{n} \frac{2r + 1}{r^2(r + 1)^2}$. **[4]**

(iii) Find $\displaystyle\sum_{r=2}^{\infty} \frac{2r + 1}{r^2(r + 1)^2}$. **[2]**

8 The complex number a is such that $a^2 = 5 - 12\mathrm{i}$.

(i) Use an algebraic method to find the two possible values of a. **[5]**

(ii) Sketch on a single Argand diagram the two possible loci given by $|z - a| = |a|$. **[4]**

9 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & 1 \\ 1 & 1 & a \end{pmatrix}$, where $a \neq 1$.

(i) Find $\mathbf{A}^{-1}$. **[7]**

(ii) Hence, or otherwise, solve the equations

$$\begin{aligned} 2x - y + z &= 1, \\ 3y + z &= 2, \\ x + y + az &= 2. \end{aligned}$$ **[4]**

10 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(i) Find $\mathbf{M}^2$ and $\mathbf{M}^3$. **[3]**

(ii) Hence suggest a suitable form for the matrix $\mathbf{M}^n$. **[1]**

(iii) Use induction to prove that your answer to part **(ii)** is correct. **[4]**

(iv) Describe fully the single geometrical transformation represented by $\mathbf{M}^{10}$. **[3]**