OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Further Pure Mathematics 2

4726

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Monday 11 January 2010
Morning

Duration: 1 hour 30 minutes

* 4 7 2 6 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 It is given that $f(x) = x^2 - \sin x$.

(i) The iteration $x_{n+1} = \sqrt{\sin x_n}$, with $x_1 = 0.875$, is to be used to find a real root, α, of the equation $f(x) = 0$. Find x_2, x_3 and x_4, giving the answers correct to 6 decimal places. **[2]**

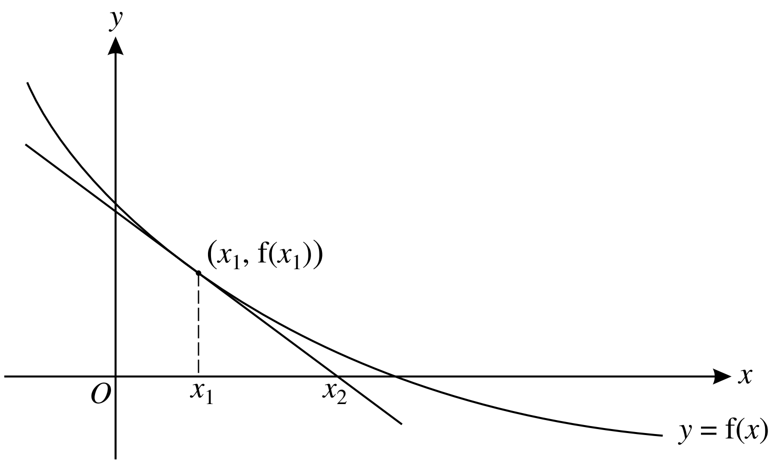
(ii) The error e_n is defined by $e_n = \alpha - x_n$. Given that $\alpha = 0.876\,726$, correct to 6 decimal places, find e_3 and e_4. Given that $g(x) = \sqrt{\sin x}$, use e_3 and e_4 to estimate $g'(\alpha)$. **[3]**

2 It is given that $f(x) = \tan^{-1}(1 + x)$.

(i) Find $f(0)$ and $f'(0)$, and show that $f''(0) = -\frac{1}{2}$. **[4]**

(ii) Hence find the Maclaurin series for $f(x)$ up to and including the term in x^2. **[2]**

3



A curve with no stationary points has equation $y = f(x)$. The equation $f(x) = 0$ has one real root α, and the Newton-Raphson method is to be used to find α. The tangent to the curve at the point $(x_1, f(x_1))$ meets the x-axis where $x = x_2$ (see diagram).

(i) Show that $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$. **[3]**

(ii) Describe briefly, with the help of a sketch, how the Newton-Raphson method, using an initial approximation $x = x_1$, gives a sequence of approximations approaching α. **[2]**

(iii) Use the Newton-Raphson method, with a first approximation of 1, to find a second approximation to the root of $x^2 - 2\sinh x + 2 = 0$. **[2]**

4 The equation of a curve, in polar coordinates, is

$$r = e^{-2\theta}, \quad \text{for } 0 \leqslant \theta \leqslant \pi.$$

(i) Sketch the curve, stating the polar coordinates of the point at which r takes its greatest value. **[2]**

(ii) The pole is O and points P and Q, with polar coordinates (r_1, θ_1) and (r_2, θ_2) respectively, lie on the curve. Given that $\theta_2 > \theta_1$, show that the area of the region enclosed by the curve and the lines OP and OQ can be expressed as $k(r_1^2 - r_2^2)$, where k is a constant to be found. **[5]**

5 **(i)** Using the definitions of $\sinh x$ and $\cosh x$ in terms of e^x and e^{-x}, show that

$$\cosh^2 x - \sinh^2 x \equiv 1.$$

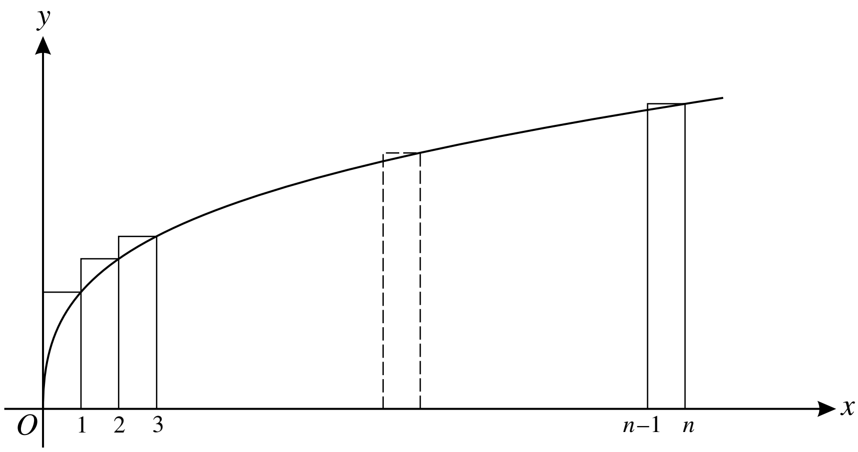
Deduce that $1 - \tanh^2 x \equiv \operatorname{sech}^2 x$. **[4]**

(ii) Solve the equation $2\tanh^2 x - \operatorname{sech} x = 1$, giving your answer(s) in logarithmic form. **[4]**

6 **(i)** Express $\dfrac{4}{(1-x)(1+x)(1+x^2)}$ in partial fractions. **[5]**

(ii) Show that $\displaystyle\int_0^{\frac{1}{\sqrt{3}}} \frac{4}{1-x^4}\,dx = \ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \tfrac{1}{3}\pi$. **[4]**

7



The diagram shows the curve with equation $y = \sqrt[3]{x}$, together with a set of n rectangles of unit width.

(i) By considering the areas of these rectangles, explain why

$$\sqrt[3]{1} + \sqrt[3]{2} + \sqrt[3]{3} + \ldots + \sqrt[3]{n} > \int_0^n \sqrt[3]{x}\,dx.$$ **[2]**

(ii) By drawing another set of rectangles and considering their areas, show that

$$\sqrt[3]{1} + \sqrt[3]{2} + \sqrt[3]{3} + \ldots + \sqrt[3]{n} < \int_1^{n+1} \sqrt[3]{x}\,dx.$$ **[3]**

(iii) Hence find an approximation to $\displaystyle\sum_{n=1}^{100} \sqrt[3]{n}$, giving your answer correct to 2 significant figures. **[3]**

[Questions 8 and 9 are printed overleaf.]

8 The equation of a curve is

$$y = \frac{kx}{(x-1)^2},$$

where k is a positive constant.

(i) Write down the equations of the asymptotes of the curve. **[2]**

(ii) Show that $y \geqslant -\frac{1}{4}k$. **[4]**

(iii) Show that the x-coordinate of the stationary point of the curve is independent of k, and sketch the curve. **[4]**

9 **(i)** Given that $y = \tanh^{-1} x$, for $-1 < x < 1$, prove that $y = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$. **[3]**

(ii) It is given that $f(x) = a\cosh x - b\sinh x$, where a and b are positive constants.

(a) Given that $b \geqslant a$, show that the curve with equation $y = f(x)$ has no stationary points. **[3]**

(b) In the case where $a > 1$ and $b = 1$, show that $f(x)$ has a minimum value of $\sqrt{a^2 - 1}$. **[6]**