

**ADVANCED GCE**  
**MATHEMATICS**  
Further Pure Mathematics 2

**4726**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

None

**Monday 11 January 2010**  
**Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 It is given that  $f(x) = x^2 - \sin x$ .

(i) The iteration  $x_{n+1} = \sqrt{\sin x_n}$ , with  $x_1 = 0.875$ , is to be used to find a real root,  $\alpha$ , of the equation  $f(x) = 0$ . Find  $x_2$ ,  $x_3$  and  $x_4$ , giving the answers correct to 6 decimal places. [2]

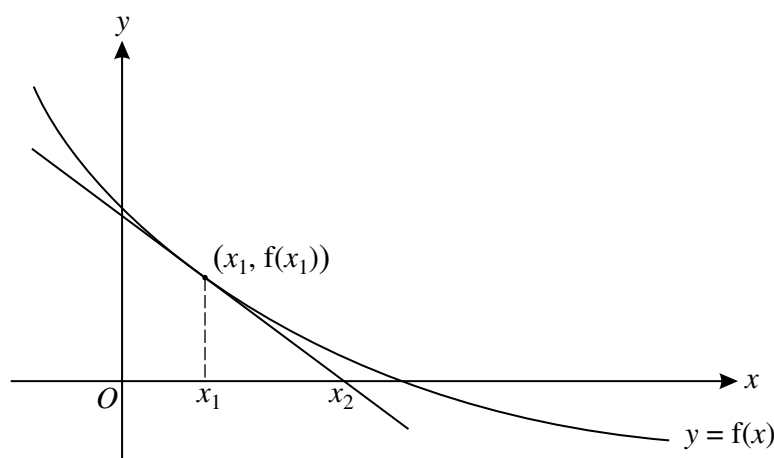
(ii) The error  $e_n$  is defined by  $e_n = \alpha - x_n$ . Given that  $\alpha = 0.876\,726$ , correct to 6 decimal places, find  $e_3$  and  $e_4$ . Given that  $g(x) = \sqrt{\sin x}$ , use  $e_3$  and  $e_4$  to estimate  $g'(\alpha)$ . [3]

2 It is given that  $f(x) = \tan^{-1}(1 + x)$ .

(i) Find  $f(0)$  and  $f'(0)$ , and show that  $f''(0) = -\frac{1}{2}$ . [4]

(ii) Hence find the Maclaurin series for  $f(x)$  up to and including the term in  $x^2$ . [2]

3



A curve with no stationary points has equation  $y = f(x)$ . The equation  $f(x) = 0$  has one real root  $\alpha$ , and the Newton-Raphson method is to be used to find  $\alpha$ . The tangent to the curve at the point  $(x_1, f(x_1))$  meets the  $x$ -axis where  $x = x_2$  (see diagram).

(i) Show that  $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ . [3]

(ii) Describe briefly, with the help of a sketch, how the Newton-Raphson method, using an initial approximation  $x = x_1$ , gives a sequence of approximations approaching  $\alpha$ . [2]

(iii) Use the Newton-Raphson method, with a first approximation of 1, to find a second approximation to the root of  $x^2 - 2 \sinh x + 2 = 0$ . [2]

4 The equation of a curve, in polar coordinates, is

$$r = e^{-2\theta}, \quad \text{for } 0 \leq \theta \leq \pi.$$

(i) Sketch the curve, stating the polar coordinates of the point at which  $r$  takes its greatest value. [2]

(ii) The pole is  $O$  and points  $P$  and  $Q$ , with polar coordinates  $(r_1, \theta_1)$  and  $(r_2, \theta_2)$  respectively, lie on the curve. Given that  $\theta_2 > \theta_1$ , show that the area of the region enclosed by the curve and the lines  $OP$  and  $OQ$  can be expressed as  $k(r_1^2 - r_2^2)$ , where  $k$  is a constant to be found. [5]

- 5 (i) Using the definitions of  $\sinh x$  and  $\cosh x$  in terms of  $e^x$  and  $e^{-x}$ , show that

$$\cosh^2 x - \sinh^2 x \equiv 1.$$

Deduce that  $1 - \tanh^2 x \equiv \operatorname{sech}^2 x$ .

[4]

- (ii) Solve the equation  $2 \tanh^2 x - \operatorname{sech} x = 1$ , giving your answer(s) in logarithmic form.

[4]

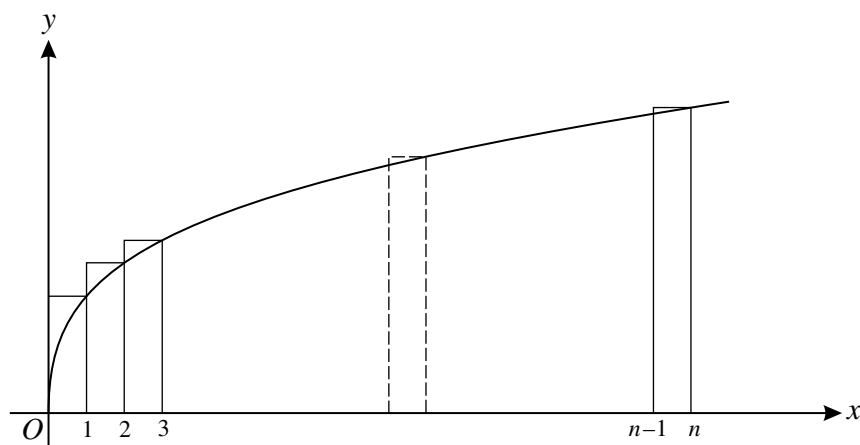
- 6 (i) Express  $\frac{4}{(1-x)(1+x)(1+x^2)}$  in partial fractions.

[5]

- (ii) Show that  $\int_0^{\frac{1}{\sqrt{3}}} \frac{4}{1-x^4} dx = \ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{1}{3}\pi$ .

[4]

7



The diagram shows the curve with equation  $y = \sqrt[3]{x}$ , together with a set of  $n$  rectangles of unit width.

- (i) By considering the areas of these rectangles, explain why

$$\sqrt[3]{1} + \sqrt[3]{2} + \sqrt[3]{3} + \dots + \sqrt[3]{n} > \int_0^n \sqrt[3]{x} dx.$$

[2]

- (ii) By drawing another set of rectangles and considering their areas, show that

$$\sqrt[3]{1} + \sqrt[3]{2} + \sqrt[3]{3} + \dots + \sqrt[3]{n} < \int_1^{n+1} \sqrt[3]{x} dx.$$

[3]

- (iii) Hence find an approximation to  $\sum_{n=1}^{100} \sqrt[3]{n}$ , giving your answer correct to 2 significant figures.

[3]

[Questions 8 and 9 are printed overleaf.]

8 The equation of a curve is

$$y = \frac{kx}{(x-1)^2},$$

where  $k$  is a positive constant.

(i) Write down the equations of the asymptotes of the curve. [2]

(ii) Show that  $y \geq -\frac{1}{4}k$ . [4]

(iii) Show that the  $x$ -coordinate of the stationary point of the curve is independent of  $k$ , and sketch the curve. [4]

9 (i) Given that  $y = \tanh^{-1} x$ , for  $-1 < x < 1$ , prove that  $y = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right)$ . [3]

(ii) It is given that  $f(x) = a \cosh x - b \sinh x$ , where  $a$  and  $b$  are positive constants.

(a) Given that  $b \geq a$ , show that the curve with equation  $y = f(x)$  has no stationary points. [3]

(b) In the case where  $a > 1$  and  $b = 1$ , show that  $f(x)$  has a minimum value of  $\sqrt{a^2 - 1}$ . [6]

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