

ADVANCED GCE MATHEMATICS Further Pure Mathematics 3

4727

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required: None

Friday 29 January 2010 Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer all the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is **72**.
- This document consists of 4 pages. Any blank pages are indicated.

1 Determine whether the lines

$$\frac{x-1}{1} = \frac{y+2}{-1} = \frac{z+4}{2} \quad \text{and} \quad \frac{x+3}{2} = \frac{y-1}{3} = \frac{z-5}{4}$$
w.
[5]

intersect or are skew.

- 2 *H* denotes the set of numbers of the form $a + b\sqrt{5}$, where *a* and *b* are rational. The numbers are combined under multiplication.
 - (i) Show that the product of any two members of *H* is a member of *H*. [2]

It is now given that, for a and b not both zero, H forms a group under multiplication.

- (ii) State the identity element of the group. [1]
- (iii) Find the inverse of $a + b\sqrt{5}$. [2]
- (iv) With reference to your answer to part (iii), state a property of the number 5 which ensures that every number in the group has an inverse. [1]
- 3 Use the integrating factor method to find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = \mathrm{e}^{-3x}$$

for which y = 1 when x = 0. Express your answer in the form y = f(x). [6]

4 (i) Write down, in cartesian form, the roots of the equation $z^4 = 16$. [2]

(ii) Hence solve the equation $w^4 = 16(1 - w)^4$, giving your answers in cartesian form. [5]

5 A regular tetrahedron has vertices at the points

$$A(0, 0, \frac{2}{3}\sqrt{6}), \quad B(\frac{2}{3}\sqrt{3}, 0, 0), \quad C(-\frac{1}{3}\sqrt{3}, 1, 0), \quad D(-\frac{1}{3}\sqrt{3}, -1, 0).$$

(i) Obtain the equation of the face ABC in the form

$$x + \sqrt{3}y + \left(\frac{1}{2}\sqrt{2}\right)z = \frac{2}{3}\sqrt{3}.$$
 [5]

(Answers which only verify the given equation will not receive full credit.)

(ii) Give a geometrical reason why the equation of the face ABD can be expressed as

$$x - \sqrt{3}y + (\frac{1}{2}\sqrt{2})z = \frac{2}{3}\sqrt{3}.$$
 [2]

(iii) Hence find the cosine of the angle between two faces of the tetrahedron. [4]

6 The variables *x* and *y* satisfy the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 16y = 8\cos 4x$$

- (i) Find the complementary function of the differential equation.
- (ii) Given that there is a particular integral of the form $y = px \sin 4x$, where p is a constant, find the general solution of the equation. [6]
- (iii) Find the solution of the equation for which y = 2 and $\frac{dy}{dx} = 0$ when x = 0. [4]
- 7 (i) Solve the equation $\cos 6\theta = 0$, for $0 < \theta < \pi$. [3]
 - (ii) By using de Moivre's theorem, show that

$$\cos 6\theta \equiv (2\cos^2\theta - 1)(16\cos^4\theta - 16\cos^2\theta + 1).$$
 [5]

(iii) Hence find the exact value of

$$\cos\left(\frac{1}{12}\pi\right)\cos\left(\frac{5}{12}\pi\right)\cos\left(\frac{7}{12}\pi\right)\cos\left(\frac{11}{12}\pi\right),$$
[5]

[2]

justifying your answer.

- 8 The function f is defined by $f: x \mapsto \frac{1}{2-2x}$ for $x \in \mathbb{R}$, $x \neq 0$, $x \neq \frac{1}{2}$, $x \neq 1$. The function g is defined by g(x) = ff(x).
 - (i) Show that $g(x) = \frac{1-x}{1-2x}$ and that gg(x) = x. [4]

It is given that f and g are elements of a group *K* under the operation of composition of functions. The element e is the identity, where $e : x \mapsto x$ for $x \in \mathbb{R}$, $x \neq 0$, $x \neq \frac{1}{2}$, $x \neq 1$.

- (ii) State the orders of the elements f and g. [2]
- (iii) The inverse of the element f is denoted by h. Find h(x). [2]
- (iv) Construct the operation table for the elements e, f, g, h of the group *K*. [4]



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