

Mathematics

Advanced GCE

Unit **4725**: Further Pure Mathematics 1

Mark Scheme for January 2011

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1	(i) $\begin{pmatrix} 7 & 9 \end{pmatrix}$	B1B1 2	Each element correct SC (7,9) scores B1
	(ii) (18)	B1* depB1 2	Obtain correct value Clearly given as a matrix
	(iii) $\begin{pmatrix} 12 & -4 \\ 6 & -2 \end{pmatrix}$	M1 A1 A1 3 $\boxed{7}$	Obtain 2×2 matrix Obtain 2 correct elements Obtain other 2 correct elements
2.	(i) $-12 + 13i$	B1B1 2	Real and imaginary parts correct
	(ii) $\frac{27}{37} - \frac{14}{37}i$	B1 M1 A1 A1 4 $\boxed{6}$	z^* seen Multiply by w^* Obtain correct real part or numerator Obtain correct imaginary part or denom. Sufficient working must be shown
3		B1* M1* A1* depA1 4 $\boxed{4}$	Establish result true for $n = 1$ or 2 Use given result in recurrence relation in a relevant way Obtain $2^n + 1$ correctly Specific statement of induction conclusion
4	<i>Either</i> $\frac{a}{4}n^2(n+1)^2 + \frac{bn}{2}(n+1)$ $a = 4 \quad b = -4$ <i>Or</i> $a + b = 0 \quad 4a + b = 12$ $a = 4 \quad b = -4$	B1 M1 A1 M1 A1 A1 6 M1 A1 A1 M1 A1 A1 $\boxed{6}$	Correct value for $\sum r$ stated or used Express as sum of two series Obtain correct unsimplified answer Compare coefficients or substitute values for n Obtain correct answers Use 2 values for n Obtain correct equations Solve simultaneous equations Obtain correct answers
5	A^2	B1 M1 A1cao 3 $\boxed{3}$	$(A^{-1})^{-1} = A$ seen or implied Use product inverse correctly Obtain correct answer

6	(i) (a)	B1*	Vertical line
	(b)	depB1 2	Clearly through (4, 0)
		B1	Sloping line with +ve slope
		B1	Through (0, -2)
		B1ft 3	Half line starting on y-axis 45° shown convincingly
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	(ii)	B1ft	Shaded to left of their (i) (a)
		B1ft	Shaded below their (i) (b) must be +ve slope
		B1ft 3	Shaded above horizontal through their (0, -2)
			NB These 3 marks are independent, but 3/3 only for fully correct answer.

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7	(i) $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$	B1 B1 2	Each column correct
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	(ii)	B1*	Enlargement or stretch in <i>x</i> and <i>y</i> axes
		depB1 2	Scale factor $\sqrt{3}$
<hr/>			
	(iii) (a)	B1	(2,0), (6,2) indicated
		B1	(8, 2) seen
		B1 3	Accurate diagram, including unit square
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	(b) $\det C = 4$	B1	Correct value found
		B1 2	Scale factor for area

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8 (i) *Either*

$$\alpha + \beta = \frac{1}{2}, \alpha\beta = \frac{3}{2}$$

$$\alpha + \beta + \frac{\alpha + \beta}{\alpha\beta} \text{ or } \alpha + \beta + \frac{2}{3}(\alpha + \beta)$$

$$p = \frac{5}{6}$$

Or

$$3u^2 - u + 2 (= 0)$$

$$p = \frac{5}{6}$$

B1	State or use both correct results in (i) or (ii)
M1	Express sum of new roots in terms of $\alpha + \beta$ and $\alpha\beta$
M1	Substitute their values into their expression
A1 4	Obtain given answer correctly
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B1	Substitute $x = \frac{1}{u}$ and obtain correct quadratic (equation)
M1	Use sum of roots of new equation
M1	Substitute their values into their expression
A1	Obtain given answer correctly

(ii) $\alpha' \beta' = \alpha\beta + \frac{1}{\alpha\beta} + \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$

B1 Correct expansion

$$\frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

M1 Show how to deal with $\alpha^2 + \beta^2$

A1 Obtain correct expression

$$q = \frac{1}{3}$$

M1 Substitute their values into $\alpha'\beta'$

A1 5 Obtain correct answer a.e.f.

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9 (i)

$$\det \mathbf{M} = a^2 - 7a + 6$$

M1 Show correct expansion process for 3 x 3

M1 Correct evaluation of any 2 x 2

A1 3 correct answer

(ii)

M1 Solve $\det \mathbf{M} = 0$

$$a = 1 \text{ or } 6$$

A1A1 3 Obtain correct answer, ft their (i)

(iii)

M1 Attempt to eliminate one variable

A1 Obtain 2 correct equations in 2 unknowns

A1 3 Justify infinite number of solutions

SC 3/3 if unique solution conclusion consistent with their (i) or (ii)

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10 (i)

M1 Use correct denominator

A1 2 Obtain **given** answer correctly

(ii)

M1 Express terms as differences using (i)

M1 Do this for at least 3 terms

A1 First 3 terms all correct

A1 Last 2 terms all correct

$$\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$$

M1 Show relevant cancelling

A1 6 Obtain correct answer a.e.f.

(iii)

$$\frac{1}{2}$$

B1ft S_{∞} stated or start at $n+1$ as in (ii)

$$\frac{1}{n+1} - \frac{1}{n+2}$$

M1 S_{∞} - their (ii) or show correct cancelling

$$\frac{1}{(n+1)(n+2)}$$

A1 3 Obtain **given** answer correctly

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