



Mathematics

Advanced GCE Unit **4725:** Further Pure Mathematics 1

Mark Scheme for January 2011

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

© OCR 2011

Any enquiries about publications should be addressed to:

OCR Publications PO Box 5050 Annesley NOTTINGHAM NG15 0DL

Telephone:0870 770 6622Facsimile:01223 552610E-mail:publications@ocr.org.uk

1 (i)	(7 9)	B1B1 2	Each element correct SC (7,9) scores B1
(ii)	(18)	B1* depB1 2	Obtain correct value Clearly given as a matrix
(iii)	$\begin{pmatrix} 12 & -4 \\ 6 & -2 \end{pmatrix}$	M1	Obtain 2×2 matrix
		A1 A1 3 7	Obtain 2 correct elements Obtain other 2 correct elements
2. (i)	- 12 +13i	B1B1 2	Real and imaginary parts correct
(ii)		B1 M1	z* seen Multiply by w*
	$\frac{27}{37} - \frac{14}{37}i$	A1	Obtain correct real part or numerator
	51 51	A1 4	Obtain correct imaginary part or denom.
		6	Sufficient working must be shown
3		B1* M1*	Establish result true for $n = 1$ or 2 Use given result in recurrence relation in a relevant way
		A1* depA1 4	Obtain $2^n + 1$ correctly Specific statement of induction conclusion
		4	
4	Either	B1	Correct value for $\sum r$ stated or used
	a hu	M1	Express as sum of two series
	$\frac{a}{4}n^2(n+1)^2 + \frac{bn}{2}(n+1)$	A1	Obtain correct unsimplified answer
		M1	Compare coefficients or substitute values for <i>n</i>
	a = 4 $b = -4$	A1 A1 6	Obtain correct answers
	Or	M1	Use 2 values for <i>n</i>
	a+b=0 4a+b=12	A1 A1	Obtain correct equations
	a = 4 $b = -4$	M1 A1 A1	Solve simultaneous equations Obtain correct answers
		6	
5	\mathbf{A}^2	B1 M1 A1cao 3 3	$(\mathbf{A}^{-1})^{-1} = \mathbf{A}$ seen or implied Use product inverse correctly Obtain correct answer

6	(i)	(a) (b)	B1* depB B1 B1 B1ft	1 2 3	Vertical line Clearly through (4, 0) Sloping line with +ve slope Through (0, -2) Half line starting on y-axis 45° shown convincingly
	(ii)		B1ft B1ft B1ft 8	3	Shaded to left of their (i) (a) Shaded below their (i) (b) must be +ve slope Shaded above horizontal through their (0, -2) NB These 3 marks are independent, but 3/3 only for fully correct answer.
7	(i)	$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$	B1 B	1 2	Each column correct
	(ii)		B1* depB	1 2	Enlargement or stretch in x and y axes Scale factor $\sqrt{3}$
	(iii)	(a)	B1		(2,0),(6,2) indicated
	()		B1 B1	3	(8, 2) seen Accurate diagram, including unit square
		(b) $\det \mathbf{C} = 4$	B1 B1 9	2	Correct value found Scale factor for area
	8	(i) Either			
	U	$\alpha + \beta = \frac{1}{2}, \alpha \beta = \frac{3}{2}$	B1		State or use both correct results in (i) or (ii)
		$\alpha + \beta + \frac{\alpha + \beta}{\alpha \beta}$ or $\alpha + \beta + \frac{2}{3}(\alpha + \beta)$	M1		Express sum of new roots in terms of
					$\alpha + \beta$ and $\alpha\beta$
			M1		Substitute their values into their expression
		$p = \frac{5}{6}$	A1	4	Obtain given answer correctly
		Or			
		$3u^2 - u + 2(= 0)$	B1		Substitute $x = \frac{1}{u}$ and obtain correct
		_	M1 M1		quadratic (equation) Use sum of roots of new equation Substitute their values into their expression
		$p = \frac{5}{6}$	A1		Obtain given answer correctly

(i) $\alpha' \beta' = \alpha \beta + \frac{1}{\alpha \beta} + \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$ B1 Correct expansion $\beta + \frac{\alpha}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$ A1 Obtain correct expression $q = \frac{1}{3}$ 9 (i) $q = \frac{1}{3}$ M1 Substitute their values into $\alpha' \beta'$ A1 5 Obtain correct answer a.e.f. 9 (i) $det M = a^2 - 7a + 6$ A1 3 correct answer (ii) a = 1 or 6 A1 3 correct answer A1 3 Obtain correct answer, fi their (i) M1 Solve det M = 0 a = 1 or 6 A1 3 Obtain correct answer, fi their (i) (iii) M1 Attempt to eliminate one variable A1 3 Usity infinite number of solutions SC 3/3 if unique solution conclusion consistent with their (i) or (ii) (ii) M1 Express terms as differences using (i) M1 Solve det and correct A1 4 6 Obtain correct answer a.e.f. (iii) A1 5 Obtain correct answer a.e.f. (ii) A1 6 Obtain correct answer a.e.f. (iii) A1 6 Obtain correct answer a.e.f. (iii) $\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$ A1 6 Obtain correct answer a.e.f. (iii) $\frac{1}{n+1} - \frac{1}{n+2}$ A1 3 Obtain given answer correctly	4725		Scheme January 2011
$\frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$ M1 Show how to deal with $\alpha^2 + \beta^2$ A1 Obtain correct expression $q = \frac{1}{3}$ M1 Substitute their values into $\alpha'\beta'$ A1 5 Obtain correct answer a.e.f. 9 (i) M1 Show correct expansion process for 3 x 3 M1 Correct answer (ii) M1 Solve det M = 0 A1 3 correct answer (iii) M1 Attempt to eliminate or variable (iii) M1 Attempt to eliminate or variable (iii) M1 Attempt to eliminate or variable (iii) M1 Attempt to eliminate (iii) M1 Attempt explored explored (iii) M1 Attempt explored explored (iii) M1 Attempt explored (iii) M1 Atte	(ii)	$\alpha' \beta' = \alpha \beta + \frac{1}{\alpha \beta} + \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$	B1 Correct expansion
$q = \frac{1}{3}$ M1Substitute their values into $\alpha'\beta'$ A19(i)M1Show correct answer a.e.f.9(i)M1Show correct evaluation of any 2 x 2 A1det $M = a^2 - 7a + 6$ A13correct answer(ii)M1Solve det M = 0 A1A1A1 $a = 1 \text{ or } 6$ A1A1Obtain correct answer, ft their (i)(iii)M1Attempt to eliminate one variable A1A1Obtain 2 correct quations in 2 unknowns A1Justifinite number of solutions SC 3/3 if unique solution conclusion consistent with their (i) or (ii)10(i)M1(ii)M1Use correct dominator A1(iii)M1Luse correct dominator A1(iii)M1Luse correct dominator A1(iii)M1Express terns a differences using (i) Do this for at leas 3 terms A1(iii)M1Show relevant cancelling A1 $\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$ B1ft $\frac{1}{n+1} - \frac{1}{n+2}$ M1M1 S_w stated or start at $n+1$ as in (ii) $M1$ $\frac{1}{(n+1)(n+2)}$ A13Obtain given answer correctly		$\frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$	M1 Show how to deal with $\alpha^2 + \beta^2$
A15Obtain correct answer a.e.f.9(i)M1Show correct expansion process for 3 x 3 M1det M = $a^2 - 7a + 6$ A13correct evaluation of any 2 x 2 A1(ii)M1Solve det M = 0 $a = 1 \text{ or } 6$ A1A13Obtain correct answer(iii)M1Solve det M = 0 $a = 1 \text{ or } 6$ A1A13Obtain correct answer, ft their (i)(iii)M1Attempt to eliminate one variable Obtain 2 correct equations in 2 unknowns A1A13Justify infinite number of solutions SC 3/3 ft urique solution conclusion consistent with their (i) or (ii)10(i)M1(ii)M1Express terms as differences using (i) M1M1Show relevant cancelling A1A1416Obtain correct answer a.e.f.(iii) $\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$ M1Show relevant cancelling A1 $\frac{1}{(n+1)(n+2)}$ A1A13Obtain given answer correctly		, ,	A1 Obtain correct expression
9(i)M1 M1 Correct evaluation of any 2 x 2det M = $a^2 - 7a + 6$ A13correct answer(ii)M1Solve det M = 0 $a = 1 \text{ or } 6$ A1A1 3Obtain correct answer(iii)M1Attempt to eliminate one variable0A1Obtain 2 correct quations in 2 unknownsA13Justify infinite number of solutions soft and their (i) or (ii)10M1Attempt to eliminate one variable Obtain 2 correct quations in 2 unknowns A110M1Attempt to eliminate one variable Obtain 2 correct quations in 2 unknowns A110M1Attempt to eliminate one variable Obtain 2 correct denominator A110M1Luse correct denominator A110M1Express terms as differences using (i) M1 Do this for at least 3 terms A1 First 3 terms all correct A1 Last 2 terms all correct(iii) $\frac{1}{2}$ $\frac{1}{n+1} - \frac{1}{n+2}$ B1ft M1 S _m - their (ii) or show correct cancelling A1 $\frac{1}{(n+1)(n+2)}$ A13Obtain given answer correctly		$q = \frac{1}{3}$	M1 Substitute their values into $\alpha'\beta'$
M1Correct evaluation of any $2 \ge 2$ (ii)M1Solve det $M = 0$ $a = 1 \text{ or } 6$ A1A1 3Obtain correct answer(iii)M1Solve det $M = 0$ $a = 1 \text{ or } 6$ A1A1 3Obtain correct answer, ft their (i)(iii)M1Attempt to climinate one variableA13Justify infinite number of solutions SC 3/3 if unique solution conclusion consistent with their (i) or (ii)10(i)M1Use correct denominator A1(ii)M1Express terms as differences using (i) Do this for at least 3 terms A1(iii)M1Express terms as differences using (i) M1 Do this for at least 3 terms A1(iii)M1Show relevant cancelling A1 $\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$ M1 M1 M1(iii) $\frac{1}{2}$ B1ft S_{∞} stated or start at $n + 1$ as in (ii) $M1$ $S_{\infty} - $ their (ii) or show correct cancelling $A1 - 1 + 1 + 2$ $\frac{1}{(n+1)(n+2)}$ A13			
(ii) M1 Solve det M = 0 $a = 1 \text{ or } 6$ A1A1 3 Obtain correct answer, ft their (i) (iii) M1 Attempt to eliminate one variable A1 3 Justify infinite number of solutions SC 3/3 if unique solution conclusion SC 3/3 if unique solution conclusion consistent with their (i) or (ii) M1 Use correct denominator A1 2 Obtain given answer correctly (ii) M1 Express terms as differences using (i) M1 Do this for at least 3 terms A1 First 3 terms all correct A1 Last 2 terms all correct $\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$ M1 Show relevant cancelling A1 6 0btain correct answer a.c.f. (iii) $\frac{1}{2}$ $\frac{1}{n+1} - \frac{1}{n+2}$ M1 $A1 = 3$ Obtain given answer correctly	9 (i)		
(ii)M1Solve det M = 0 $a = 1 \text{ or } 6$ A1A1 3Obtain correct answer, ft their (i)(iii)M1Attempt to eliminate one variable A1A13Justify infinite number of solutions SC 3/3 if unique solution conclusion consistent with their (i) or (ii)10M1Use correct denominator A1(ii)M1Express terms as differences using (i) Do this for at least 3 terms A1(ii)M1Express terms all correct A1 $\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$ M1Show relevant cancelling A1(iii) $\frac{1}{2}$ B1ft S_{∞} stated or start at $n + 1$ as in (ii) $\frac{1}{(n+1)(n+2)}$ A13Obtain given answer correctly			A1 3 correct answer
(iii) M1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1 A1	(ii)		M1 Solve det $\mathbf{M} = 0$
(iii) M1 Attempt to eliminate one variable Obtain 2 correct equations in 2 unknowns A1 3 Justify infinite number of solutions SC 3/3 if unique solution conclusion consistent with their (i) or (ii) 10 (i) M1 Use correct denominator A1 2 Obtain given answer correctly (ii) M1 Express terms as differences using (i) M1 Do this for at least 3 terms A1 First 3 terms all correct A1 Last 2 terms all correct A1 Last 2 terms all correct A1 6 Obtain correct answer a.e.f. (iii) $\frac{1}{2}$ $\frac{1}{n+1} - \frac{1}{n+2}$ $\frac{1}{(n+1)(n+2)}$ A1 3 Obtain given answer correctly			
10 (i) M1 Use correct denominator (ii) M1 Express terms as differences using (i) (iii) M1 Express terms as differences using (i) M1 Do this for at least 3 terms A1 2 (iii) M1 $\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$ M1 Show relevant cancelling A1 6 Obtain correct answer a.e.f. (iii) $\frac{1}{2}$ $\frac{1}{n+1} - \frac{1}{n+2}$ M1 Show relevant at $n + 1$ as in (ii) $\frac{1}{(n+1)(n+2)}$ M1 A1 3 Obtain given answer correctly	(iii)		 M1 Attempt to eliminate one variable A1 Obtain 2 correct equations in 2 unknowns A1 3 Justify infinite number of solutions SC 3/3 if unique solution conclusion
A12Obtain given answer correctly(ii)M1Express terms as differences using (i) Do this for at least 3 terms A1 Express terms all correct A1 Express terms all correct $\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$ M1Show relevant cancelling A1 A1(iii) $\frac{1}{2}$ $\frac{1}{n+1} - \frac{1}{n+2}$ B1ft M1 S _{\infty} stated or start at $n + 1$ as in (ii) $\frac{1}{(n+1)(n+2)}$ A13Obtain given answer correctly			9
(ii) $M1 = Express terms as differences using (i) M1 = Do this for at least 3 terms A1 = First 3 terms all correct A1 = Last 2 terms all correct and A1 = 6 = Obtain correct answer a.e.f. (iii) \frac{1}{2} A1 = 6 = Obtain correct answer a.e.f.(iii) \frac{1}{2} B1ft = S_{\infty} stated or start at n + 1 as in (ii) M1 = S_{\infty} - their (ii) or show correct cancelling M1 = S_{\infty} - their (ii) or show correct cancelling \frac{1}{(n+1)(n+2)} A1 = 3 = Obtain given answer correctly$	10 (i)		A1 2 Obtain given answer correctly
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	(ii)		M1Express terms as differences using (i)M1Do this for at least 3 termsA1First 3 terms all correct
A16Obtain correct answer a.e.f.(iii) $\frac{1}{2}$ B1ft S_{∞} stated or start at $n + 1$ as in (ii) $\frac{1}{n+1} - \frac{1}{n+2}$ M1 S_{∞} - their (ii) or show correct cancelling $\frac{1}{(n+1)(n+2)}$ A13Obtain given answer correctly		$\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$	M1 Show relevant cancelling
$\frac{1}{n+1} - \frac{1}{n+2}$ M1 S_{∞} - their (ii) or show correct cancelling $\frac{1}{(n+1)(n+2)}$ A1 3 Obtain given answer correctly		2 n+1 n+2	A1 6 Obtain correct answer a.e.f.
$\frac{1}{(n+1)(n+2)}$ M1 S_{∞} - their (ii) or show correct cancelling A1 3 Obtain given answer correctly	(iii)	$\frac{1}{2}$	B1ft S_{∞} stated or start at $n + 1$ as in (ii)
(n+1)(n+2)		_	M1 S_{∞} - their (ii) or show correct cancelling
· · · · · · · · · · · · · · · · · · ·		$\frac{1}{(\ldots,1)(\ldots,2)}$	A1 3 Obtain given answer correctly
<u> 11 </u>		(n+1)(n+2)	11

OCR (Oxford Cambridge and RSA Examinations) 1 Hills Road Cambridge CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)

Telephone: 01223 553998 Facsimile: 01223 552627 Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations is a Company Limited by Guarantee Registered in England Registered Office; 1 Hills Road, Cambridge, CB1 2EU Registered Company Number: 3484466 OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations) Head office Telephone: 01223 552552 Facsimile: 01223 552553

