

# Mathematics

Advanced GCE

Unit 4727: Further Pure Mathematics 3

## Mark Scheme for January 2011

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1 (i)	Integrating factor. $e^{\int x dx} = e^{\frac{1}{2}x^2}$	B1	For correct IF
	$\Rightarrow \frac{d}{dx} \left( y e^{\frac{1}{2}x^2} \right) = x e^{x^2}$	M1	For $\frac{d}{dx} (y \cdot \text{their IF}) = x e^{\frac{1}{2}x^2} \cdot \text{their IF}$
	$\Rightarrow y e^{\frac{1}{2}x^2} = \frac{1}{2} e^{x^2} (+c)$	A1	For correct integration both sides
	$\Rightarrow y = e^{-\frac{1}{2}x^2} \left( \frac{1}{2} e^{x^2} + c \right) = \frac{1}{2} e^{\frac{1}{2}x^2} + c e^{-\frac{1}{2}x^2}$	A1 4	For correct solution <b>AEF</b> as $y = f(x)$
(ii)	$(0, 1) \Rightarrow c = \frac{1}{2}$	M1	For substituting (0, 1) into their GS, solving for $c$ and obtaining a solution of the DE
	$\Rightarrow y = \frac{1}{2} \left( e^{\frac{1}{2}x^2} + e^{-\frac{1}{2}x^2} \right)$	A1 2	For correct solution <b>AEF</b> Allow $y = \cosh\left(\frac{1}{2}x^2\right)$
<b>6</b>			
2 (i)	$\mathbf{n} = [2, 1, -3] \times [-1, 2, 4]$	M1	For using $\times$ of direction vectors
	$= [10, -5, 5] = k[2, -1, 1]$	A1	For correct $\mathbf{n}$
	$(1, 3, 4) \Rightarrow 2x - y + z = 3$	A1 3	For substituting (1, 3, 4) and obtaining <b>AG</b> (Verification only M0)
(ii)	METHOD 1	M1	For $21 - 3$ OR $[1, 3, 4] \cdot [2, -1, 1] - 21$
	distance = $\frac{21-3}{ \mathbf{n} }$ OR $\frac{[1, 3, 4] \cdot [2, -1, 1] - 21}{ \mathbf{n} }$		OR $ (1, 3, 4) - [a, b, c] \cdot [2, -1, 1] $ soi
	OR $\frac{ (1, 3, 4) - [a, b, c] \cdot [2, -1, 1] }{ \mathbf{n} }$ where $(a, b, c)$ is on $q$	B1	For $ \mathbf{n}  = \sqrt{6}$ soi
	$= \frac{18}{\sqrt{6}} = 3\sqrt{6}$	A1 3	For correct distance <b>AEF</b>
METHOD 2	$[1+2t, 3-t, 4+t]$ on $q$	M1	For forming and solving an equation in $t$
	$\Rightarrow 2(1+2t) - (3-t) + (4+t) = 21 \Rightarrow t = 3$	B1	For $ \mathbf{n}  = \sqrt{6}$ soi
	$\Rightarrow \text{distance} = 3 \mathbf{n}  = 3\sqrt{6}$	A1	For correct distance <b>AEF</b>
METHOD 3	As Method 2 to $t = 3 \Rightarrow (7, 0, 7)$ on $q$	M1*	For finding point where normal meets $q$
	distance from (1, 3, 4)	M1	For finding distance from (1, 3, 4)
	$= \sqrt{(7-1)^2 + (0-3)^2 + (7-4)^2} = \sqrt{54} = 3\sqrt{6}$	(*dep) A1	For correct distance <b>AEF</b>
<b>6</b>			
3 (i)	$\sin \theta = \frac{1}{2i} (e^{i\theta} - e^{-i\theta})$	B1	$z$ or $e^{i\theta}$ may be used throughout For correct expression for $\sin \theta$ soi
	$\sin^4 \theta = \frac{1}{16} (z^4 - 4z^2 + 6 - 4z^{-2} + z^{-4})$	M1	For expanding $(e^{i\theta} - e^{-i\theta})^4$ (with at least 3 terms and 1 binomial coefficient)
	$\Rightarrow \sin^4 \theta = \frac{1}{16} (2 \cos 4\theta - 8 \cos 2\theta + 6)$	M1	For grouping terms and using multiple angles
	$\Rightarrow \sin^4 \theta = \frac{1}{8} (\cos 4\theta - 4 \cos 2\theta + 3)$	A1 4	For answer obtained correctly <b>AG</b>
(ii)	$\int_0^{\frac{1}{6}\pi} \sin^4 \theta d\theta = \frac{1}{8} \left[ \frac{1}{4} \sin 4\theta - 2 \sin 2\theta + 3\theta \right]_0^{\frac{1}{6}\pi}$	M1	For integrating (i) to $A \sin 4\theta + B \sin 2\theta + C\theta$
		A1	For correct integration
	$= \frac{1}{8} \left( \frac{1}{8} \sqrt{3} - \sqrt{3} + \frac{1}{2} \pi \right) = \frac{1}{64} (4\pi - 7\sqrt{3})$	M1	For completing integration and substituting limits
		A1 4	For correct answer <b>AEF</b> (exact)
<b>8</b>			

<p>4 (i)</p>	<p><i>EITHER</i> <math>1 + \omega + \omega^2</math>  <math>=</math> sum of roots of <math>(z^3 - 1) = 0</math></p> <hr/> <p>OR <math>\omega^3 = 1 \Rightarrow (\omega - 1)(\omega^2 + \omega + 1) = 0</math>  <math>\Rightarrow 1 + \omega + \omega^2 = 0</math> (for <math>\omega \neq 1</math>)</p> <hr/> <p>OR sum of G.P.  <math>1 + \omega + \omega^2 = \frac{1 - \omega^3}{1 - \omega} \left( = \frac{0}{1 - \omega} \right) = 0</math></p> <hr/> <p>OR  shown on Argand diagram  or explained in terms of  vectors</p> <hr/> <p>OR  <math>1 + \text{cis } \frac{2}{3}\pi + \text{cis } \frac{4}{3}\pi = 1 + \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 0</math></p>	<p>M1  A1 2</p>	<p>For result shown by any correct method <b>AG</b></p>
<p>(ii)</p>	<p>Multiplication by <math>\omega \Rightarrow</math> rotation through <math>\frac{2}{3}\pi</math> </p> <p><math>z_1 - z_3 = \vec{CA}</math>, <math>z_3 - z_2 = \vec{BC}</math></p> <p><math>\vec{BC}</math> rotates through <math>\frac{2}{3}\pi</math> to direction of <math>\vec{CA}</math></p> <p><math>\Delta ABC</math> has <math>BC = CA</math>, hence result</p>	<p>B1  B1  M1  A1 4</p>	<p>For correct interpretation of <math>\times</math> by <math>\omega</math>  (allow <math>120^\circ</math> and omission of, or error in, <math>\odot</math>)</p> <p>For identification of vectors soi  (ignore direction errors)</p> <p>For linking <math>BC</math> and <math>CA</math> by rotation of <math>\frac{2}{3}\pi</math> OR <math>\omega</math></p> <p>For stating equal magnitudes <math>\Rightarrow</math> <b>AG</b></p>
<p>(iii)</p>	<p>(ii) <math>\Rightarrow z_1 + \omega z_2 - (1 + \omega)z_3 = 0</math></p> <p><math>1 + \omega + \omega^2 = 0 \Rightarrow z_1 + \omega z_2 + \omega^2 z_3 = 0</math></p>	<p>M1  A1 2</p>	<p>For using <math>1 + \omega + \omega^2 = 0</math> in (ii)</p> <p>For obtaining <b>AG</b></p>
<p><b>8</b></p>			
<p>5 (i)</p>	<p>Aux. equation <math>3m^2 + 5m - 2 (= 0)</math></p> <p><math>\Rightarrow m = \frac{1}{3}, -2</math></p> <p>CF (<math>y =</math>) <math>Ae^{\frac{1}{3}x} + Be^{-2x}</math></p> <p>PI (<math>y =</math>) <math>px + q \Rightarrow 5p - 2(px + q) = -2x + 13</math></p> <p><math>\Rightarrow p = 1, q = -4</math></p> <p>GS (<math>y =</math>) <math>Ae^{\frac{1}{3}x} + Be^{-2x} + x - 4</math></p>	<p>M1  A1  A1√  M1  A1 A1  B1√ 7</p>	<p>For correct auxiliary equation seen  and solution attempted</p> <p>For correct roots</p> <p>For correct CF  f.t. from <math>m</math> with 2 arbitrary constants</p> <p>For stating and substituting PI of correct form</p> <p>For correct value of <math>p</math>, and of <math>q</math></p> <p>For GS  f.t. from their CF+PI with 2 arbitrary constants  in CF and none in PI</p>
<p>(ii)</p>	<p><math>\left(0, -\frac{7}{2}\right) \Rightarrow A + B = \frac{1}{2}</math></p> <p><math>y' = \frac{1}{3}Ae^{\frac{1}{3}x} - 2Be^{-2x} + 1, (0, 0) \Rightarrow A - 6B = -3</math></p> <p><math>\Rightarrow A = 0, B = \frac{1}{2}</math></p> <p><math>\Rightarrow (y =) \frac{1}{2}e^{-2x} + x - 4</math></p>	<p>M1  M1  M1  A1  B1√ 5</p>	<p>For substituting <math>\left(0, -\frac{7}{2}\right)</math> in their GS  and obtaining an equation in <math>A</math> and <math>B</math></p> <p>For finding <math>y'</math>, substituting <math>(0, 0)</math>  and obtaining an equation in <math>A</math> and <math>B</math></p> <p>For solving their 2 equations in <math>A</math> and <math>B</math></p> <p>For correct <math>A</math> and <math>B</math> <b>CAO</b></p> <p>For correct solution  f.t. with their <math>A</math> and <math>B</math> in their GS</p>
<p>(iii)</p>	<p><math>x</math> large <math>\Rightarrow (y =) x - 4</math></p>	<p>B1√ 1</p>	<p>For correct equation or function  (allow <math>\approx</math> and <math>\rightarrow</math>) <b>WWW</b>  f.t. from (ii) if valid</p>
<p><b>13</b></p>			

6 (i)	$a^4 = r^6 = e \Rightarrow a$ has order 4, $a^2$ has order 2 $(a^3)^4 = a^{12} = e \Rightarrow a^3$ has order 4 $(r^2)^3 = e \Rightarrow r^2$ has order 3	M1	For considering powers of $a$										
		A1	For order of any one of $a, a^2, a^3$ correct										
		A1	For all correct										
		B1	4 For order of $r^2$ correct										
(ii)	<b>G order 4</b> <table border="1" data-bbox="261 412 740 477"> <tr> <td>Order of element</td> <td>1</td> <td>2</td> <td>(4)</td> </tr> <tr> <td>Number of elements</td> <td>1</td> <td>3</td> <td>(0)</td> </tr> </table>	Order of element	1	2	(4)	Number of elements	1	3	(0)	M1	For top line in either table Allow inclusion of 4 and 6 respectively (and other orders if 0 appears below)		
Order of element	1	2	(4)										
Number of elements	1	3	(0)										
	<b>H order 6</b> <table border="1" data-bbox="261 506 815 573"> <tr> <td>Order of element</td> <td>1</td> <td>2</td> <td>3</td> <td>(6)</td> </tr> <tr> <td>Number of elements</td> <td>1</td> <td>3</td> <td>2</td> <td>(0)</td> </tr> </table>	Order of element	1	2	3	(6)	Number of elements	1	3	2	(0)	A1	For order 4 table
Order of element	1	2	3	(6)									
Number of elements	1	3	2	(0)									
		A1	For order 6 table										
	G and H are the only non-cyclic groups of order which divides 12	B1	For stating that only G and H need be considered <b>AEF</b>										
	Q has 1 element of order 2, G and H have 3, so no non-cyclic subgroups in Q	B1	5 For argument completed by elements of order 2 <b>AG</b> <b>SR</b> Allow equivalent arguments for B1 B1										
9													
7 (i)	$[1, 1, -2] \times [1, -1, 3] = (\pm)[1, -5, -2]$ $[1, -1, 3] \times [1, 5, -12] = (\pm)[-3, 15, 6]$ $[-3, 15, 6] = k [1, -5, -2] \Rightarrow$ parallel	M1	For using $\times$ of direction vectors										
		A1	For correct direction										
		M1	For using $\times$ of direction vectors										
		A1	For correct direction										
		A1	5 For argument completed <b>AG</b> ( $k = -3$ not essential)										
(ii)	Line of intersection is parallel to $l$ and $m$	B1	1 For correct statement										
(iii)	METHOD 1												
	$\left. \begin{matrix} x + y - 2z = 5 \\ x - y + 3z = 6 \end{matrix} \right\}$ e.g. $z = 0 \Rightarrow \left(\frac{11}{2}, -\frac{1}{2}, 0\right)$ on $l$	M1	For attempt to find points on 2 lines										
		A1	For a correct point on one line										
	$\left. \begin{matrix} x - y + 3z = 6 \\ x + 5y - 12z = 12 \end{matrix} \right\}$ e.g. $z = 0 \Rightarrow (7, 1, 0)$ on $m$	A1	For a correct point on another line										
	$\left. \begin{matrix} x + y - 2z = 5 \\ x + 5y - 12z = 12 \end{matrix} \right\}$ e.g. $z = 0 \Rightarrow \left(\frac{13}{4}, \frac{7}{4}, 0\right)$ on $l_3$												
	Different points $\Rightarrow$ no common line of intersection	A1	4 For correct answer										
	METHOD 2												
	$\left. \begin{matrix} x + y - 2z = 5 \\ x - y + 3z = 6 \end{matrix} \right\}$ e.g. $\Rightarrow z = 11 - 2x, y = 27 - 5x$	M1	For finding (e.g.) $y$ and $z$ in terms of $x$ OR eliminating one variable										
	LHS of eqn 3 =	A1	For correct expressions OR equations										
	$x + (135 - 25x) - (132 - 24x) = 3 \neq 12$	A1	For obtaining a contradiction from 3rd equation										
	$\Rightarrow$ no common line of intersection	A1	For correct answer										
	METHOD 3												
	LHS $II_3 = 3II_1 - 2II_2$	M2	For attempt to link 3 equations										
	RHS $3 \times 5 - 2 \times 6 = 3 \neq 12$	A1	For obtaining a contradiction										
	$\Rightarrow$ no common line of intersection	A1	For correct answer										
	<b>SR</b> Variations on all methods may gain full credit		<b>SR</b> f.t. may be allowed from relevant working										
10													

<b>8 (i)</b>	$((a,b)*(c,d))*(e,f) = (ac, ad+b)*(e,f)$	M1	For 3 distinct elements bracketed and attempt to expand
	$= (ace, acf + ad + b)$	A1	For correct expression
	$(a,b)*((c,d)*(e,f)) = (a,b)*(ce, cf + d)$ $= (ace, acf + ad + b)$	A1	<b>3</b> For correct expression again
<b>(ii)</b>	$(a,b)*(1,1) = (a, a+b), (1,1)*(a,b) = (a, b+1)$	M1	For combining both ways round
	$a+b = b+1 \Rightarrow a = 1$	M1	For equating components (allow from incorrect pairs)
	$\Rightarrow (1, b) \forall b$	A1	<b>3</b> For correct elements <b>AEF</b>
<b>(iii)</b>	$(mp, mq+n) \text{ OR } (pm, pn+q) = (1, 0)$	M1	For either element on LHS
	$\Rightarrow (p, q) = \left(\frac{1}{m}, -\frac{n}{m}\right)$	A1	<b>2</b> For correct inverse
<b>(iv)</b>	$(a,b)*(a,b) = (a^2, ab+b) = (1, 0)$	M1	For attempt to find self-inverses
	$\text{OR } (a,b) = \left(\frac{1}{a}, -\frac{b}{a}\right) \Rightarrow a^2 = 1, ab = -b$	B1	For (1, 0). For (-1, b) <b>AEF</b>
	$\Rightarrow$ self-inverse elements (1, 0) and $(-1, b) \forall b$	A1	<b>3</b>
<b>(v)</b>	$(0, y)$ has no inverse for any $y \Rightarrow$ not a group	B1	<b>1</b> For stating any one element with no inverse. Allow $x \neq 0$ required, provided reference to inverse is made "Some elements have no inverse" B0

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