OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4725

Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4725
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Wednesday 19 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 The matrices **A**, **B** and **C** are given by $\mathbf{A} = (2 \quad 5)$, $\mathbf{B} = (3 \quad -1)$ and $\mathbf{C} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$. Find

(i) $2\mathbf{A} + \mathbf{B}$, **[2]**

(ii) $\mathbf{AC}$, **[2]**

(iii) $\mathbf{CB}$. **[3]**

2 The complex numbers z and w are given by $z = 4 + 3\mathrm{i}$ and $w = 6 - \mathrm{i}$. Giving your answers in the form $x + \mathrm{i}y$ and showing clearly how you obtain them, find

(i) $3z - 4w$, **[2]**

(ii) $\dfrac{z^*}{w}$. **[4]**

3 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 2$, and $u_{n+1} = 2u_n - 1$ for $n \geqslant 1$. Prove by induction that $u_n = 2^{n-1} + 1$. **[4]**

4 Given that $\sum_{r=1}^{n}(ar^3 + br) \equiv n(n-1)(n+1)(n+2)$, find the values of the constants a and b. **[6]**

5 Given that **A** and **B** are non-singular square matrices, simplify

$$\mathbf{AB}(\mathbf{A}^{-1}\mathbf{B})^{-1}.$$ **[3]**

6 **(i)** Sketch on a single Argand diagram the loci given by

(a) $|z| = |z - 8|$, **[2]**

(b) $\arg(z + 2\mathrm{i}) = \frac{1}{4}\pi$. **[3]**

(ii) Indicate by shading the region of the Argand diagram for which

$$|z| \leqslant |z - 8| \quad \text{and} \quad 0 \leqslant \arg(z + 2\mathrm{i}) \leqslant \tfrac{1}{4}\pi.$$ **[3]**

7 **(i)** Write down the matrix, **A**, that represents a shear with x-axis invariant in which the image of the point (1, 1) is (4, 1). **[2]**

(ii) The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{3} \end{pmatrix}$. Describe fully the geometrical transformation represented by **B**. **[2]**

(iii) The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 2 & 6 \\ 0 & 2 \end{pmatrix}$.

(a) Draw a diagram showing the unit square and its image under the transformation represented by **C**. **[3]**

(b) Write down the determinant of **C** and explain briefly how this value relates to the transformation represented by **C**. **[2]**

8 The quadratic equation $2x^2 - x + 3 = 0$ has roots α and β, and the quadratic equation $x^2 - px + q = 0$ has roots $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$.

(i) Show that $p = \frac{5}{6}$. **[4]**

(ii) Find the value of q. **[5]**

9 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} a & -a & 1 \\ 3 & a & 1 \\ 4 & 2 & 1 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of $\mathbf{M}$. **[3]**

(ii) Hence find the values of a for which $\mathbf{M}^{-1}$ does not exist. **[3]**

(iii) Determine whether the simultaneous equations

$$\begin{aligned} 6x - 6y + z &= 3k, \\ 3x + 6y + z &= 0, \\ 4x + 2y + z &= k, \end{aligned}$$

where k is a non-zero constant, have a unique solution, no solution or an infinite number of solutions, justifying your answer. **[3]**

10 **(i)** Show that $\frac{1}{r} - \frac{2}{r+1} + \frac{1}{r+2} \equiv \frac{2}{r(r+1)(r+2)}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)}.$$ **[6]**

(iii) Show that $\sum_{r=n+1}^{\infty} \frac{2}{r(r+1)(r+2)} = \frac{1}{(n+1)(n+2)}$. **[3]**

There are no questions printed on this page.

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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1 (i)	
1 (ii)	
1 (iii)	

2 (i)

2 (ii)

3	

4

5	
6	

7 (i)	
7 (ii)	
7 (iii) (a)	
7 (iii) (b)	

8 (i)	
8 (ii)	

9 (i)

9 (ii)

9 (iii)	
10 (i)	

10 (ii)	

10 (iii)	