

**ADVANCED GCE
MATHEMATICS**

Further Pure Mathematics 2

4726

Candidates answer on the answer booklet.

OCR supplied materials:

- 8 page answer booklet
(sent with general stationery)
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

**Monday 10 January 2011
Morning**

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Use the substitution $t = \tan \frac{1}{2}x$ to find $\int \frac{1}{1 + \sin x + \cos x} dx$. [5]

2 It is given that $f(x) = \tanh^{-1} x$.

(i) Show that $f'''(x) = \frac{2(1 + 3x^2)}{(1 - x^2)^3}$. [5]

(ii) Hence find the Maclaurin series for $f(x)$, up to and including the term in x^3 . [3]

3 The function f is defined by $f(x) = \frac{5ax}{x^2 + a^2}$, for $x \in \mathbb{R}$ and $a > 0$.

(i) For the curve with equation $y = f(x)$,

(a) write down the equation of the asymptote, [1]

(b) find the range of values that y can take. [4]

(ii) For the curve with equation $y^2 = f(x)$, write down

(a) the equation of the line of symmetry, [1]

(b) the maximum and minimum values of y , [2]

(c) the set of values of x for which the curve is defined. [1]

4 (i) Use the definitions of hyperbolic functions in terms of exponentials to prove that

$$8 \sinh^4 x \equiv \cosh 4x - 4 \cosh 2x + 3. \quad [4]$$

(ii) Solve the equation

$$\cosh 4x - 3 \cosh 2x + 1 = 0,$$

giving your answer(s) in logarithmic form. [5]

5 The equation

$$x^3 - 5x + 3 = 0 \quad (\text{A})$$

may be solved by the Newton-Raphson method. Successive approximations to a root are denoted by $x_1, x_2, \dots, x_n, \dots$

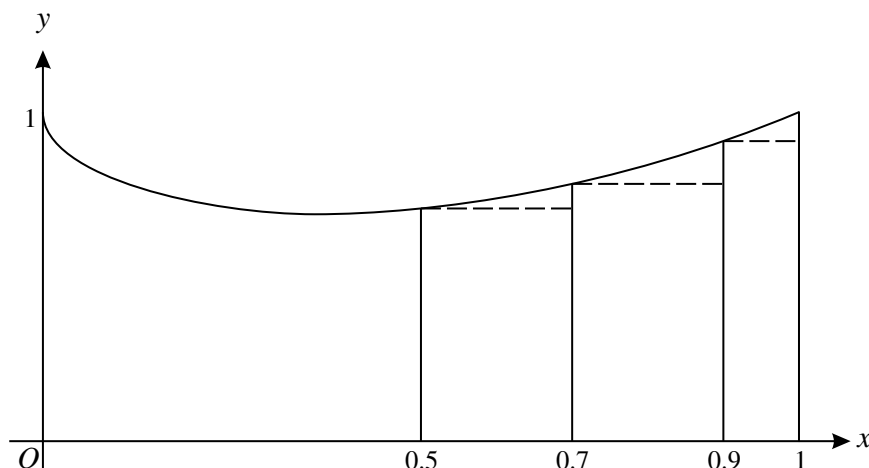
(i) Show that the Newton-Raphson formula can be written in the form $x_{n+1} = F(x_n)$, where

$$F(x) = \frac{2x^3 - 3}{3x^2 - 5}. \quad [3]$$

(ii) Find $F'(x)$ and hence verify that $F'(\alpha) = 0$, where α is any one of the roots of equation (A). [3]

(iii) Use the Newton-Raphson method to find the root of equation (A) which is close to 2. Write down sufficient approximations to find the root correct to 4 decimal places. [3]

6



The diagram shows the curve $y = f(x)$, defined by

$$f(x) = \begin{cases} x^x & \text{for } 0 < x \leq 1, \\ 1 & \text{for } x = 0. \end{cases}$$

- (i) By first taking logarithms, show that the curve has a stationary point at $x = e^{-1}$. [3]

The area under the curve from $x = 0.5$ to $x = 1$ is denoted by A .

- (ii) By considering the set of three rectangles shown in the diagram, show that a lower bound for A is 0.388. [2]
- (iii) By considering another set of three rectangles, find an upper bound for A , giving 3 decimal places in your answer. [2]

The area under the curve from $x = 0$ to $x = 0.5$ is denoted by B .

- (iv) Draw a diagram to show rectangles which could be used to find lower and upper bounds for B , using not more than three rectangles for each bound. (You are not required to find the bounds.) [3]

7 A curve has polar equation $r = 1 + \cos 3\theta$, for $-\pi < \theta \leq \pi$.

- (i) Show that the line $\theta = 0$ is a line of symmetry. [2]
- (ii) Find the equations of the tangents at the pole. [3]
- (iii) Find the exact value of the area of the region enclosed by the curve between $\theta = -\frac{1}{3}\pi$ and $\theta = \frac{1}{3}\pi$. [5]

8 (i) Without using a calculator, show that $\sinh(\cosh^{-1} 2) = \sqrt{3}$. [2]

(ii) It is given that, for non-negative integers n ,

$$I_n = \int_0^\beta \cosh^n x \, dx, \quad \text{where } \beta = \cosh^{-1} 2.$$

Show that $nI_n = 2^{n-1}\sqrt{3} + (n-1)I_{n-2}$, for $n \geq 2$. [6]

- (iii) Evaluate I_5 , giving your answer in the form $k\sqrt{3}$. [4]

There are no questions printed on this page.

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