

**ADVANCED GCE
MATHEMATICS**

Further Pure Mathematics 3

4727

Candidates answer on the answer booklet.

OCR supplied materials:

- 8 page answer booklet
(sent with general stationery)
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

**Friday 28 January 2011
Morning**

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

- 1 (i) Find the general solution of the differential equation

$$\frac{dy}{dx} + xy = xe^{\frac{1}{2}x^2},$$

giving your answer in the form $y = f(x)$.

[4]

- (ii) Find the particular solution for which $y = 1$ when $x = 0$.

[2]

- 2 Two intersecting lines, lying in a plane p , have equations

$$\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{-3} \quad \text{and} \quad \frac{x-1}{-1} = \frac{y-3}{2} = \frac{z-4}{4}.$$

- (i) Obtain the equation of p in the form $2x - y + z = 3$.

[3]

- (ii) Plane q has equation $2x - y + z = 21$. Find the distance between p and q .

[3]

- 3 (i) Express $\sin \theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$ and show that

$$\sin^4 \theta \equiv \frac{1}{8}(\cos 4\theta - 4 \cos 2\theta + 3).$$

[4]

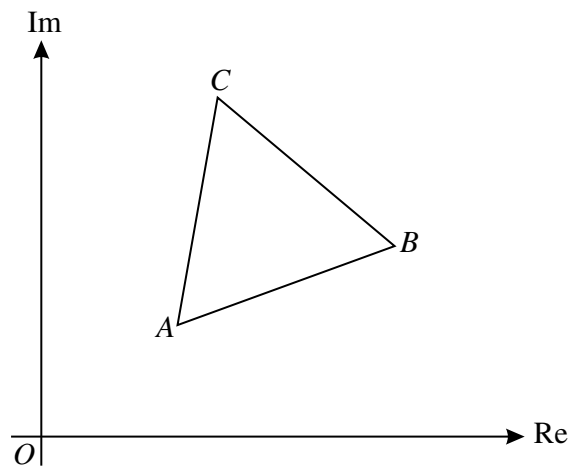
- (ii) Hence find the exact value of $\int_0^{\frac{1}{6}\pi} \sin^4 \theta \, d\theta$.

[4]

- 4 The cube roots of 1 are denoted by 1, ω and ω^2 , where the imaginary part of ω is positive.

- (i) Show that $1 + \omega + \omega^2 = 0$.

[2]



In the diagram, ABC is an equilateral triangle, labelled anticlockwise. The points A , B and C represent the complex numbers z_1 , z_2 and z_3 respectively.

- (ii) State the geometrical effect of multiplication by ω and hence explain why $z_1 - z_3 = \omega(z_3 - z_2)$.

[4]

- (iii) Hence show that $z_1 + \omega z_2 + \omega^2 z_3 = 0$.

[2]

- 5 (i) Find the general solution of the differential equation

$$3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = -2x + 13. \quad [7]$$

- (ii) Find the particular solution for which $y = -\frac{7}{2}$ and $\frac{dy}{dx} = 0$ when $x = 0$. [5]

- (iii) Write down the function to which y approximates when x is large and positive. [1]

- 6 Q is a multiplicative group of order 12.

- (i) Two elements of Q are a and r . It is given that r has order 6 and that $a^2 = r^3$. Find the orders of the elements a , a^2 , a^3 and r^2 . [4]

The table below shows the number of elements of Q with each possible order.

| | | | | | |
|--------------------|---|---|---|---|---|
| Order of element | 1 | 2 | 3 | 4 | 6 |
| Number of elements | 1 | 1 | 2 | 6 | 2 |

G and H are the non-cyclic groups of order 4 and 6 respectively.

- (ii) Construct two tables, similar to the one above, to show the number of elements with each possible order for the groups G and H . Hence explain why there are no non-cyclic proper subgroups of Q . [5]

- 7 Three planes Π_1 , Π_2 and Π_3 have equations

$$\mathbf{r} \cdot (\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 5, \quad \mathbf{r} \cdot (\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 6, \quad \mathbf{r} \cdot (\mathbf{i} + 5\mathbf{j} - 12\mathbf{k}) = 12,$$

respectively. Planes Π_1 and Π_2 intersect in a line l ; planes Π_2 and Π_3 intersect in a line m .

- (i) Show that l and m are in the same direction. [5]
- (ii) Write down what you can deduce about the line of intersection of planes Π_1 and Π_3 . [1]
- (iii) By considering the cartesian equations of Π_1 , Π_2 and Π_3 , or otherwise, determine whether or not the three planes have a common line of intersection. [4]

[Question 8 is printed overleaf.]

8 The operation $*$ is defined on the elements (x, y) , where $x, y \in \mathbb{R}$, by

$$(a, b) * (c, d) = (ac, ad + b).$$

It is given that the identity element is $(1, 0)$.

(i) Prove that $*$ is associative. [3]

(ii) Find all the elements which commute with $(1, 1)$. [3]

(iii) It is given that the particular element (m, n) has an inverse denoted by (p, q) , where

$$(m, n) * (p, q) = (p, q) * (m, n) = (1, 0).$$

Find (p, q) in terms of m and n . [2]

(iv) Find all self-inverse elements. [3]

(v) Give a reason why the elements (x, y) , under the operation $*$, do not form a group. [1]

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