GCE

Mathematics (MEI)

Advanced GCE

Unit **4753:** Methods for Advanced Mathematics

Mark Scheme for January 2012

Oxford Cambridge and RSA Examinations

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$y = x^2 \tan 2x$	M1	product rule	u × their v' + v × their u' attempted
				M1	d/du(tan u) = $\sec^2 u$ soi	M0 if d/dx (tan 2x) =(2) $\sec^2 x$
			$\Rightarrow$ dy/dx = $2x^2\sec^2 2x + 2x\tan 2x$	A1cao	or $2x^2/\cos^2 2x + 2x\tan 2x$	isw
			OR $y = x^2 \dfrac{\sin 2x}{\cos 2x}$			*see additional notes for complete solution*
			$\dfrac{dy}{dx} = x^2 \dfrac{\cos 2x.2\cos 2x - \sin 2x(-2\sin 2x)}{\cos^2 2x} + 2x\dfrac{\sin 2x}{\cos 2x}$	M1	product rule	u × their v' + v × their u' attempted
				A1	correct expression	
			= … = $2x^2\sec^2 2x + 2x\tan 2x$	A1cao	or $2x^2/\cos^2 2x + 2x\tan 2x$ (isw)	or $(2x^2 + 2x\sin 2x\cos 2x)/\cos^2 2x$
						or $2x^2/\cos^2 2x + 2x\sin 2x / \cos 2x$
			OR $y = \dfrac{x^2 \sin 2x}{\cos 2x}$			*see additional notes for complete solution*
			$\dfrac{dy}{dx} = \dfrac{\cos 2x(2x\sin 2x + x^2 2\cos 2x) - 2x^2 \sin 2x(-\sin 2x)}{\cos^2 2x}$	M1	quotient rule	(v × their u' − u × their v')/v^2 attempted
				A1	correct expression	
			= … = $2x^2\sec^2 2x + 2x\tan 2x$	A1cao	or $2x^2/\cos^2 2x + 2x\tan 2x$ (isw)	
				[3]		or $(2x^2 + 2x\sin 2x\cos 2x)/\cos^2 2x$
						or $2x^2/\cos^2 2x + 2x\sin 2x / \cos 2x$
2			fg(x) = $\ln(1+x^2)$ ($x \in \Re$)	B1	condone missing bracket, and	If fg and gf the wrong way round, B1B0
			gf(x) = $1+(\ln x)^2$ ($x > 0$)	B1	missing or incorrect domains	not $1 + \ln(x^2)$
			$\ln(1+x^2)$ even	B1	Penalise missing bracket	
			$1 + (\ln x)^2$ neither	B1	Penalise missing bracket	
				[4]		
3			$u = x$, du/dx = 1, dv/dx = cos ½ x, v = 2sin ½ x	M1	correct u, u', v, v'	but allow v to be any multiple of sin ½ x
						M0 if u = cos ½ x, , v' = x
			$\int_0^{\pi/2} x\cos\frac{1}{2}x\,dx = \left[2x\sin\frac{1}{2}x\right]_0^{\pi/2} - \int_0^{\pi/2} 2\sin\frac{1}{2}x\,dx$	A1ft	consistent with their u, v	
			$= \left[2x\sin\frac{1}{2}x + 4\cos\frac{1}{2}x\right]_0^{\pi/2}$	A1	$2x$ sin ½ x + 4 cos ½ x oe (no ft)	
			$= \pi\sin\frac{\pi}{4} + 4\cos\frac{\pi}{4} - (2.0.\sin 0 + 4\cos 0)$	M1	substituting correct limits into correct expression	can be implied by one correct intermediate step
			$= \pi.\frac{1}{\sqrt{2}} + 4.\frac{1}{\sqrt{2}} - 4$			
			$= \frac{\sqrt{2}}{2}\pi + 2\sqrt{2} - 4$*	A1cao	**NB AG**	
				[5]		

4			Cubes are 1, 8, 27, 64, 125, 216, 343, 512 [so false as] $8^3 = 512$	M1 A1 **[2]**	Attempt to find counter example counter-example identified (e.g. underlining, circling) [counter-examples all have 8 as units digit]	if no counter-example found, award M1 if at least 3 cubes are calculated. condone not explicitly stating statement is false
5	(i)		(One-way) stretch in y-direction, s.f. 2 or in x-direction s.f. ½ translation 1 to right (2 if followed by x-stretch) $y = 2\|x-1\|$	 B1 B1 B1 **[3]**	 must specify s.f. and direction o.e. e.g. $y = \|2x-2\|$ $y = \|2(x-1)\|$	Allow 'compress', 'squeeze'(for s.f. ½), but not 'enlarge', 'x-coordinates halved', etc Allow 'shift','move' or vector only, 'right 1' Don't allow misreads (e.g. transforming solid graph to dashed graph) Award B1 for one of these seen, and a second B1 if combined transformations are correct
5	(ii)		Reflection in x-axis or translation right ±π or rotation of 180° [about O] translation +1 in y-direction (− 1 if followed by reflection in x-axis $y = 1 - \cos x$	 B1 B1 B1 **[3]**	 $\begin{pmatrix} \pm\pi \\ 1 \end{pmatrix}$ is B2 allow $1 + \cos(x \pm \pi)$ (bracket needed)	Translations as above. Reflection: must specify axis, allow 'flip' Rotation: condone no origin stated. *See additional notes for other possible solutions*. Award B1 for any one of these seen, and a second B1 if combined transformations are correct
6	(i)		When $t = 2$, $r = 20(1 - e^{-0.4}) = 6.59$ m $dr/dt = -20 \times (-0.2e^{-0.2t})$ $= 4e^{-0.2t}$ When $t = 2$, $dr/dt = 2.68$	M1A1 M1 A1 **[4]**	6.6 or art 6.59 $-0.2e^{-0.2t}$ soi 2.7 or art 2.68 or $4e^{-0.4}$	 mark final answer
6	(ii)		$A = \pi r^2$ $\Rightarrow$ $dA/dr = 2\pi r$ (= 41.428…) $dA/dt = (dA/dr) \times (dr/dt)$ = 41.428… × 2.68 = 111 m^2/hr	M1 A1 M1 A1 **[4]**	attempt to differentiate πr^2 $dA/dr = 2\pi r$ (not dA/dt, dr/dA etc) (o.e.) chain rule expressed in terms of their A, r or implied 110 or art 111	or differentiating $400\pi(1 - e^{-0.2t})^2$ M1 $dA/dt = 400\pi.2(1 - e^{-0.2t}).(-0.2e^{-0.2t})$ A1 substitute $t = 2$ into correct dA/dt M1 (Could use another letter for A)

7	(i)		$x^3 + y^3 = 3xy$ $\Rightarrow \quad 3x^2 + 3y^2(\mathrm{d}y/\mathrm{d}x) = 3x(\mathrm{d}y/\mathrm{d}x) + 3y$ $\Rightarrow \quad (3y^2 - 3x)(\mathrm{d}y/\mathrm{d}x) = 3y - 3x^2$ $\Rightarrow \quad \mathrm{d}y/\mathrm{d}x = (3y - 3x^2)/(3y^2 - 3x)$ $= (y - x^2)/(y^2 - x)$*	B1B1 M1 A1cao [4]	LHS, RHS Condone $3x\mathrm{d}y/\mathrm{d}x+y$ (i.e.with missing bracket) if recovered thereafter collecting terms in $\mathrm{d}y/\mathrm{d}x$ and factorising NB **AG**	or equivalent if re-arranged. ft correct algebra on incorrect expressions with two $\mathrm{d}y/\mathrm{d}x$ terms Ignore starting with '$\mathrm{d}y/\mathrm{d}x = \ldots$' unless pursued
7	(ii)		TP when $y - x^2 = 0$ $\Rightarrow \quad y = x^2$ $\Rightarrow \quad x^3 + x^6 = 3x.x^2$ $\Rightarrow \quad x^6 = 2x^3$ $\Rightarrow \quad x^3 = 2$ (or $x = 0$) $\Rightarrow \quad x = \sqrt[3]{2}$	M1 M1 A1 A1cao [4]	or $x = \sqrt{y}$ substituting for y in implicit eqn (allow one slip, e.g. x^5) o.e. (soi) must be exact	or x for y (i.e. $y^{3/2} + y^3 = 3y^{1/2}y$ o.e.) or $y^{3/2} = 2$ $x = 1.2599\ldots$ is A0 (but can isw $x = \sqrt[3]{2}$)
8	(i)		When $x = 3$, $y = 3/\sqrt{(3-2)} = 3$ So P is (3, 3) which lies on $y = x$	M1 A1 **[2]**	substituting $x = 3$ (both x's) $y = 3$ and completion ('3 = 3' is enough)	or $x = x/\sqrt{(x-2)}$ M1 $\Rightarrow x = 3$ A1(by solving or verifying)
8	(ii)		$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sqrt{x-2}.1 - x.\frac{1}{2}.(x-2)^{-1/2}}{x-2}$ $= \dfrac{x-2-\frac{1}{2}x}{(x-2)^{3/2}} = \dfrac{\frac{1}{2}x-2}{(x-2)^{3/2}}$ $= \dfrac{x-4}{2(x-2)^{3/2}}$* When $x = 3$, $\mathrm{d}y/\mathrm{d}x = -\tfrac{1}{2} \times 1^{3/2}$ $= -\tfrac{1}{2}$ This gradient would be –1 if curve were symmetrical about $y = x$	M1 A1 M1 A1 M1 A1 A1cao **[7]**	Quotient or product rule PR: $-\tfrac{1}{2}x(x-2)^{-3/2} + (x-2)^{-1/2}$ correct expression $\times$ top and bottom by $\sqrt{(x-2)}$ o.e. e.g. taking out factor of $(x-2)^{-3/2}$ NB **AG** substituting $x = 3$ or an equivalent valid argument	If correct formula stated, allow one error; otherwise QR must be on correct u and v, with numerator consistent with their derivatives and denominator correct initially allow ft on correct equivalent algebra from their incorrect expression

8	**(iii)**		$u = x - 2 \Rightarrow du/dx = 1 \Rightarrow du = dx$ When $x = 3$, $u = 1$ when $x = 11$, $u = 9$ $\Rightarrow \int_3^{11} \frac{x}{\sqrt{x-2}}\,dx = \int_1^9 \frac{u+2}{u^{1/2}}\,du$ $= \int_1^9 (u^{1/2} + 2u^{-1/2})\,du$ $= \left[\frac{2}{3}u^{3/2} + 4u^{1/2}\right]_1^9$ $= (18 + 12) - (2/3 + 4)$ $= 25\frac{1}{3}$ * Area under $y = x$ is ½ (3 + 11) × 8 = 56 Area = (area under $y = x$) – (area under curve) so required area $= 56 - 25\frac{1}{3} = 30\frac{2}{3}$	B1 B1 M1 A1 M1 A1cao B1 M1 A1cao **[9]**	or $dx/du = 1$ $\int \frac{u+2}{u^{1/2}}(du)$ splitting their fraction (correctly) and $u/u^{1/2} = u^{1/2}$ (or $\sqrt{u}$) $\left[\frac{2}{3}u^{3/2} + 4u^{1/2}\right]$ (o.e) substituting correct limits NB AG o.e. (e.g. 60.5 – 4.5) soi from working 30.7 or better	No credit for integrating initial integral by parts. Condone $du = 1$.Condone missing du's in subsequent working. or integration by parts: $2u^{1/2}(u+2) - \int 2u^{1/2}du$ (must be fully correct – condone missing bracket by parts: $[2u^{1/2}(u+2) - 4u^{3/2}/3]$ F(9) – F(1) (u) or F(11) – F(3) (x) dep substitution and integration attempted must be trapezium area: $60.5 - 25\frac{1}{3}$ is M0
9	**(i)**		When $x = 1$, f(1) =ln(2/2)= ln 1 = 0 so P is (1, 0) f(2) = ln(4/3)	B1 B1 **[2]**	or $\ln(2x/1+x) = 0 \Rightarrow 2x/(1+x) = 1 \Rightarrow 2x = 1+x \Rightarrow x = 1$	if approximated, can isw after ln(4/3)
9	**(ii)**		$y = \ln(2x) - \ln(1 + x)$ $\Rightarrow \frac{dy}{dx} = \frac{2}{2x} - \frac{1}{1+x}$ **OR** $\frac{d}{dx}\left(\frac{2x}{1+x}\right) = \frac{(1+x)2 - 2x.1}{(1+x)^2} = \frac{2}{(1+x)^2}$ $\frac{dy}{dx} = \frac{2}{(1+x)^2}.\frac{1}{2x/(1+x)} = \frac{1}{x(1+x)}$ At P, $dy/dx = 1 - ½ = ½$	M1 M1 A1cao B1 M1 A1 A1cao **[4]**	one term correct mark final ans correct quotient or product rule chain rule attempted o.e., but mark final ans	condone lack of brackets $2/2x$ or $-1/(1+x)$ need not be simplified need not be simplified

9	(iii)		$x = \ln[2y/(1+y)]$ or $\Rightarrow e^x = 2y/(1+y)$ $\Rightarrow e^x(1+y) = 2y$ $\Rightarrow e^x = 2y - e^x y = y(2 - e^x)$ $\Rightarrow y = e^x/(2 - e^x)$ [= g(x)] **OR** gf(x)=g(2x/(1+x)) = $e^{\ln[2x/(1+x)]}/\{2-e^{\ln[2x/(1+x)]}\}$ $= \frac{2x/(1+x)}{2-2x/(1+x)}$ $= \frac{2x}{2+2x-2x} = \frac{2x}{2} = x$ gradient at R = 1/ ½ = 2	 B1 B1 B1 B1 M1 A1 M1A1 B1 ft **[5]**	($x \leftrightarrow y$ here or at end to complete) completion forming gf or fg 1/their ans in (ii) unless ±1 or 0	$x = e^y/(2 - e^y)$ $x(2 - e^y) = e^y$ B1 $2x = e^y + xe^y = e^y(1 + x)$ B1 $2x/(1+x) = e^y$ B1 $\ln[2x/(1+x)] = y$ [= f(x)] B1 fg(x) = $\ln\{2e^x/(2-e^x)/[1+ e^x/(2-e^x)]\}$ M1 $= \ln[2e^x/(2 - e^x + e^x)]$ A1 $= \ln(e^x) = x$ M1A1 2 must follow ½ for 9(ii) unless g′(x) used (*see additional notes*)
9	(iv)		let $u = 2 - e^x \Rightarrow du/dx = -e^x$ $x = 0$, $u = 1$, $x = \ln(4/3)$, $u = 2 - 4/3 = 2/3$ $\Rightarrow \int_0^{\ln(4/3)} g(x)\,dx = \int_1^{2/3} -\frac{1}{u}\,du$ $= [-\ln(u)]_1^{2/3} = -\ln(2/3) + \ln 1 = \ln(3/2)$* Shaded region = rectangle – integral = 2ln(4/3) – ln(3/2) = ln(16/9 × 2/3) = ln(32/27)*	B1 M1 A1 A1cao M1 B1 A1cao **[7]**	$2-e^0 = 1$, and $2 - e^{\ln(4/3)} = 2/3$ seen $\int -1/u\,du$ condone $\int 1/u\,du$ [–ln(u)] (could be [ln$u$] if limits swapped) NB AG rectangle area = 2ln(4/3) NB AG must show at least one step from 2ln(4/3) – ln(3/2)	here or later (i.e. after substituting 0 and ln(4/3) into $\ln(2 - e^x)$) or by inspection [$k \ln(2 - e^x)$] $k = -1$ Allow full marks here for correctly evaluating $\int_1^2 \ln(\frac{2x}{1+x})\,dx$ (*see additional notes)*

Additional notes and solutions

1. $y = x^2 \dfrac{\sin 2x}{\cos 2x}$ $\dfrac{dy}{dx} = x^2 \dfrac{\cos 2x.2\cos 2x - \sin 2x(-2\sin 2x)}{\cos^2 2x} + 2x\dfrac{\sin 2x}{\cos 2x} = x^2 \dfrac{2\cos^2 2x + 2\sin^2 2x}{\cos^2 2x} + 2x\dfrac{\sin 2x}{\cos 2x}$

$= x^2 \dfrac{2}{\cos^2 2x} + 2x\dfrac{\sin 2x}{\cos 2x} = 2x^2 \sec^2 2x + 2x\tan 2x$

$y = \dfrac{x^2 \sin 2x}{\cos 2x}$ $\dfrac{dy}{dx} = \dfrac{\cos 2x(2x\sin 2x + x^2 2\cos 2x) - 2x^2 \sin 2x(-\sin 2x)}{\cos^2 2x}$

$= \dfrac{2x\cos 2x\sin 2x + 2x^2\cos^2 2x - x^2 \sin 2x(-2\sin^2 2x)}{\cos^2 2x} = \dfrac{2x\cos 2x\sin 2x + 2x^2\cos^2 2x + 2x^2\sin^2 2x}{\cos^2 2x}$

$= \dfrac{2x\cos 2x\sin 2x + 2x^2}{\cos^2 2x} = 2x\tan 2x + 2x^2\sec^2 x$

5 (ii) translation $\begin{pmatrix}0\\1\end{pmatrix}$ then translation $\begin{pmatrix}\pm\pi\\0\end{pmatrix}$ translation $\begin{pmatrix}\pm\pi\\0\end{pmatrix}$ then translation $\begin{pmatrix}0\\1\end{pmatrix}$ translation $\begin{pmatrix}\pm\pi\\1\end{pmatrix}$ B2

translation $\begin{pmatrix}0\\1\end{pmatrix}$ then reflection in $y = 1$ translation $\begin{pmatrix}0\\1\end{pmatrix}$ then reflection in $x = ½\,\pi$ translation $\begin{pmatrix}0\\-1\end{pmatrix}$ then reflection in x-axis

reflection in x-axis then translation $\begin{pmatrix}0\\1\end{pmatrix}$ reflection in $y = 1$ then translation $\begin{pmatrix}0\\-1\end{pmatrix}$ rotation 180° about O then translation $\begin{pmatrix}0\\1\end{pmatrix}$

reflection in $y = ½$ B2

9(iii) last part: $g(x) = e^x/(2 - e^x) \Rightarrow g'(x) = [(2 - e^x)e^x - e^x(-e^x)]/(2 - e^x)^2 = 2e^x/(2 - e^x)^2$
or $g'(x) = e^x(-1)(-e^x)](2 - e^x)^{-2} + e^x(2 - e^x)^{-1}$
$g'(0) = 2.1/1^2 = 2$ B1

9(iv) last part

$\int_1^2 \ln(\dfrac{2x}{1+x})\,dx = \int_1^2 (\ln 2 + \ln x - \ln(1+x))\,dx \quad = \left[x\ln 2 + x\ln x - x - (1+x)\ln(1+x) + x\right]_1^2$

$=2\ln 2 + 2\ln 2 - 2 - 3\ln 3 + 2 - (\ln 2 - 1 - 2\ln 2 + 1) = 5\ln 2 - 3\ln 3 = \ln(32/27)$

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