OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4733:** Probability and Statistics 2

Mark Scheme for January 2012

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1			$\hat{\mu} = \bar{x} = 3.65$	B1	3.65 stated explicitly, *not* isw
			$S^2 = \frac{739.625}{50} - 3.65^2$ [= 1.47]	M1	Correct formula for biased estimate used, award if 1.47 seen
			$\hat{\sigma}^2 = \frac{50}{49} S^2$	M1	$n/(n-1)$ factor used, or if wrong single formula, M1 if $n-1$ divisor anywhere. Correct single formula: M2
			= 1.5	A1	Answer 1.5 or exact equivalent only
				[4]	
2			Po(4.2)	M1	Po(np) stated or implied
			$e^{-4.2}\frac{4.2^5}{5!}$ **= 0.1633**	M1	Poisson formula or tables, allow for .1944, .1144, .16(0), .1663;
				A1	Answer, a.r.t. 0.163
			n large, p small	B1	One condition ⎫ Needs Poisson. If inequalities
			or $n > 50$, $np < 5$	B1	The other condition ⎭ used, must be these, but allow $p < 0.1$ if
					and nothing extraneous $n > 50$ already stated
				[5]	
3			μ = 60	B1	μ = 60 stated or implied, can be written down
			$\frac{63.8-\mu}{\sigma} = \Phi^{-1}(0.9) = 1.282$	M1	Standardise 63.8 or 56.2 with σ, allow √ or cc errors, equate to Φ^{-1}
				B1	1.282 (or 1.281 or 1.28) seen
			σ = 2.96(4)	A1	σ, in range [2.96, 2.97], can be implied by what follows, *not* σ^2
			$1-\Phi\left(\frac{65-60}{2.964}\right)$ = 1 – Φ(1.687)	M1	Standardise 65 with their μ and σ, allow √ or cc errors
			= 0.0458	A1	Final answer, a.r.t. 0.046, c.w.o.
				[6]	
4			N(2.5, 0.025)	M1	Normal (any – can be implied by standardisation)
			$\Phi\left(\frac{2.59-2.5}{\sqrt{0.025}}\right)$ = Φ(0.5692)	A1	Mean 2.5
				A1	Variance or SD 1.25 ÷ 50 stated or used
				A1	Standardise 2.59 or 2.61, with √(1.25/50)
			= **0.7154**	A1	Answer in range [0.715, 0.716] *or* [0.736, 0.737] from 0.632
				[5]	

5	(i)	(a)	${}^6C_4\ 0.6^4\ 0.4^2$ = **0.311**[04] [= 972/3125]	M1 A1 **[2]**	This formula, allow $0.6 \leftrightarrow 0.4$, or tables used correctly Final answer, exact fraction or a.r.t. 0.311
5	(i)	(b)	${}^{12}C_4 \times {}^8C_2 \div {}^{20}C_6$ [= 495×28 ÷ 38760] **= 0.3576** [= 231/646]	M1 A1 **[2]**	Product of two ${}^nC_r$ divided by ${}^nC_r$, *or* ${}^6C_2\left(\frac{12}{20}\times\frac{11}{19}\times\frac{10}{18}\times\frac{9}{17}\times\frac{8}{16}\times\frac{7}{15}\right)$ Final answer, exact fraction or a.r.t. 0.358
5	(ii)		B(60, 0.6) ≈ N(36, 14.4) $1-\Phi\left(\frac{29.5-36}{\sqrt{14.4}}\right) = 1-\Phi(-1.713)$ **= 0.9567** No effect as population is large *or* yes but not by much	B1 B1 M1 A1 A1 B1 **[6]**	N(36, …) or N(24, …); 14.4 or √14.4, both from B(60, 0.6) Standardise with their *np* and √*npq* (or *npq*) Both their √*npq* and cc correct [30.5 if using 24] Answer in range [0.956, 0.957] Need all of one of these [*not* "sample"], or equiv, nothing wrong
6			$H_0: \lambda = 6.3$ [*or* μ] $H_1: \lambda < 6.3$ [*or* μ] $P(\leq 2) = e^{-6.3}(1 + 6.3 + 19.845)$ **= 0.0498** < 0.05 Therefore reject H_0. Significant evidence that average number of pips has been reduced.	B2 M1 A1 A1 B1 M1 A1√ **[8]**	Both: B2. One error e.g. "$H_0 = 6.3$", or "$H_1: \lambda \neq 6.3$", B1, but *x*, *r* etc: 0 Correct formula for at least 2 terms, can be implied by 0.0134 Fully correct formula for ≤ 2, can be implied by answer Answer, a.r.t. 0.0498 SR tables: B2 if a.r.t. 0.0506, else 0 [then can get B1M1A1] Explicitly state < 0.05, *not* from $H_1: \lambda \neq 6.3$, *or* CR ≤ 2 and explicitly state 2 in CR, needs essentially correct distribution *Not needed for final M1A1* Correct method, comparison and first conclusion Interpreted in context acknowledging uncertainty somewhere, √ on *p* etc SR: P(< 2) [0.0134] or Po(= 2) [0.0364]: B2 M1 A0 B0 M0 but *allow* "Po(= 2) = 0.0498" etc SR: Normal: B2 M1 A0 B0

7	(i)	(a)	$\int_1^4 \frac{1}{2\sqrt{x}} x dx = \left[\frac{1}{3}x^{\frac{3}{2}}\right]_1^4$ = 7/3 or 2.333…	M1 B1 A1 **[3]**	Attempt to integrate $xf(x)$, correct limits Correct indefinite integral, a.e.f. Final answer 7/3 or equiv or a.r.t. 2.33
7	(i)	(b)	$\int_1^m \frac{1}{2\sqrt{x}} dx = 0.5$ $\sqrt{m} - 1 = 0.5$ $\boldsymbol{m = 2.25}$	M1 A1 A1 **[3]**	This or complementary integral, limits needed [not "$-\infty$"], equated to 0.5, needn't attempt to evaluate This equation, any equivalent simplified form Answer 9/4 or exact equivalent only
7	(ii)		$1.5\int_1^\infty y^{-2.5} y^2 dx = 1.5\left[\frac{y^{0.5}}{0.5}\right]_1^\infty$ Upper limit gives infinite answer	M1 B1 A1 **[3]**	Attempt to integrate $y^2f(y)$, limits 1 and ∞, allow any letter Correct indefinite integral [$=3\sqrt{y}$], ignore μ [= 3] Give correct reason, c.w.o. apart from constant, allow "$= \infty$"
8	(i)		Location of bacteria must be independent – the position of one does not affect that of another ***Examples*** α Number of bacteria occurring in a particular volume is independent of the number in another interval of the same volume. Number in one volume occurs randomly. M1A0 β Bacteria are distributed independently from one another. This means that they cannot be in groups. M1A0 γ Position of each bacterium must be independent of the position of other bacteria. Not well modelled by Poisson if they tended to form groups, they must not be influenced by the surrounding bacteria or certain conditions (e,g, heat). M1A0 δ Bacteria need to be independent. The results of one cannot influence the result of another. M1A0 ε Bacteria must occur independently, so the state of one bacterium has no effect on any other bacteria. M1A0 ς Probability of bacteria must be independent, they cannot affect the probability of another bacterium occurring. M1A1 η Bacteria must occur independently, so if one occurs it can't cause more to appear. M1A1	M1 A1 **[2]**	"Found independently": M1. Allow "are independent", "singly". Context needed somewhere in answer. Correct explanation, not just of "singly", e.g. not "must not group together". No extra or wrong conditions, but allow both "singly" and "independently". Right explanation, not "independent": M1A0

8	(ii)		$1 - P(\leq 4)$ [= 1 – 0.8912] **= 0.1088**	M1 A1 **[2]**	Allow M1 for 1 – .9580 [= 0.042] or wrong λ. 0.8912 etc: M0 0.109 or 0.1088 or better
8	(iii)		Po(0.925) $e^{-0.925}\frac{0.925^2}{2!}$ **= 0.169**(64)	M1 M1 A1 **[3]**	Po(0.925) stated or implied [37/40] Correct Po formula for $r = 2$, any λ, can be implied by: Answer 0.17(0) or 0.1696 or better
8	(iv)		Po(250) $\lambda > 15$ or λ large [or μ] N(250, 250) $\Phi\left(\frac{239.5-250}{\sqrt{250}}\right) = 1-\Phi(0.664)$ = **0.2533**	B1 B1 M1* A1√ Dep*M1 A1√ A1 **[7]**	Po(250) stated or implied Either of these N, mean their 100 × 2.5 … … variance (or SD) their mean Standardise, allow wrong or no cc and/or no √ or σ^2, needs A1 Continuity correction and √ correct Final answer a.r.t. 0.253, c.w.o.
9	(i)		H_0: $\mu = 8$; H_1: $\mu \neq 8$ where μ is the population mean amount of sleep obtained by Year 11 pupils	B2 B1 **[3]**	Both, B2. One error, B1, allow *x*/*r*/*t* here, but not $\overline{H}$ Need "population" or equivalent, but allow "average amount of sleep obtained by Year 11 pupils". Allow "μ is population mean".
9	(ii)		$\Phi\left(\frac{0.28}{0.87/\sqrt{64}}\right) = \Phi(2.575)$ 2 × (1 – above) = **0.01** or **1%**	M1 A1 M1 A1 **[4]**	Standardise, with √*n* or *n*, allow cc, √ errors *z* = 2.575 or 2.57 or 2.58, can be implied by, e.g., 0.005 or 0.995 Correct handling of tails Answer 0.01 or 1% correct to 2 SF, c.w.o.
9	(iii)		Rejecting H_0 when $\mu = 8$	B1 **[1]**	Or equivalent, some mention of context, *not* "probability of …"
9	(iv)		$\Phi\left(\frac{8.28-7.9}{0.87/\sqrt{64}}\right) - \Phi\left(\frac{7.72-7.9}{0.87/\sqrt{64}}\right)$ = Φ(3.494) – Φ(–1.655) [= 0.99976 – (1 – 0.951) or 1 – …] **= 0.951**	M1 M1 A1 **[3]**	Find P(between 7.72 and 8.28 | $\mu = 7.9$), allow 1 – 2×P(1 tail) (need attempt to find correct region, *not* isw – i.e., *not* ans 0.049) Correct handling of tails, needn't attempt to evaluate, needs 64 Final answer, a.r.t. 0.951. SR: One tail only used: M1M0A0. 0.951 from no working: B2

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