

Mathematics (MEI)

Advanced GCE

Unit **4753**: Methods for Advanced Mathematics

Mark Scheme for January 2013

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

© OCR 2013

Annotations

Annotation	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance
1	(i)	$y = e^{-x} \sin 2x$ $\Rightarrow \quad dy/dx = e^{-x} \cdot 2 \cos 2x + (-e^{-x}) \sin 2x$	M1 B1 A1 [3]	Product rule $d/dx(\sin 2x) = 2 \cos 2x$ Any correct expression $u \times \text{their } v' + v \times \text{their } u'$ but mark final answer
1	(ii)	$dy/dx = 0$ when $2 \cos 2x - \sin 2x = 0$ $\Rightarrow \quad 2 = \tan 2x$ $\Rightarrow \quad 2x = \arctan 2$ $\Rightarrow \quad x = \frac{1}{2} \arctan 2 *$	M1 M1 A1 [3]	ft their dy/dx but must eliminate e^{-x} $\sin 2x / \cos 2x = \tan 2x$ used [or \tan^{-1}] NB AG derivative must have 2 terms substituting $\frac{1}{2} \arctan 2$ into their deriv M0 (unless $\cos 2x = 1/\sqrt{5}$ and $\sin 2x = 2/\sqrt{5}$ found) must show previous step
2	(i)	$2x + 4y \frac{dy}{dx} = 4$ $\Rightarrow \quad \frac{dy}{dx} = \frac{4 - 2x}{4y}$	M1 A1 A1 [3]	$4y \frac{dy}{dx}$ correct equation o.e., but mark final answer Rearranging for y and differentiating explicitly is M0 Ignore superfluous $dy/dx = \dots$ unless used subsequently
2	(ii)	$\frac{dy}{dx} = 0 \Rightarrow x = 2$ $\Rightarrow \quad 4 + 2y^2 = 8 \Rightarrow y^2 = 2, y = \sqrt{2} \text{ or } -\sqrt{2}$	B1dep B1B1 [3]	dep correct derivative $\sqrt{2}, -\sqrt{2}$ can isw, penalise inexact answers of ± 1.41 or better once only -1 for extra solutions found from using $y = 0$
3		$1 < x < 3 \Rightarrow \quad -1 < x - 2 < 1$ $\Rightarrow \quad x - 2 < 1$	B1 B1 [2]	oe [or $a = 2$ and $b = 1$]

Question		Answer	Marks	Guidance	
4	(i)	$\theta = a - be^{-kt}$ When $t = 0, \theta = 15 \Rightarrow 15 = a - b$ When $t = \infty, \theta = 100 \Rightarrow 100 = a$ $\Rightarrow b = 85$ When $t = 1, \theta = 30 \Rightarrow 30 = 100 - 85e^{-k}$ $\Rightarrow e^{-k} = 70/85$ $\Rightarrow -k = \ln(70/85) = -0.194(156\dots)$ $\Rightarrow k = 0.194$	M1 B1 A1cao M1 M1 A1 [6]	$15 = a - b$ $a = 100$ $b = 85$ $30 = a - b e^{-k}$ Re-arranging and taking lns 0.19 or better, or $-\ln(70/85)$ oe	must have $e^0 = 1$ (need not substitute for a and b) allow $-k = \ln[(a - 30)/b]$ ft on a, b mark final ans
4	(ii)	$80 = 100 - 85 e^{-0.194t}$ $\Rightarrow e^{-0.194t} = 20/85$ $\Rightarrow t = -\ln(4/17) / 0.194 = 7.45$ (min)	M1 A1 [2]	ft their values for a, b and k art 7.5 or 7 min 30s or better	but must substitute values
5	(i)	$dF/dv = -25 v^{-2}$	M1 A1 [2]	$d/dv(v^{-1}) = -v^{-2}$ soi $-25 v^{-2}$ o.e mark final ans	
5	(ii)	When $v = 50, dF/dv = -25/50^2 (= -0.01)$ $\frac{dF}{dt} = \frac{dF}{dv} \cdot \frac{dv}{dt}$ $= -0.01 \times 1.5 = -0.015$	B1 M1 A1cao [3]	$-25/50^2$ o.e. o.e. e.g. $-3/200$ isw	e.g. $\frac{dF}{dv} = \frac{dF}{dt} / \frac{dv}{dt}$

Question	Answer	Marks	Guidance	
6	<p>Let $u = 1 + x \Rightarrow$ $\int_0^3 x(1+x)^{-1/2} dx = \int_1^4 (u-1)u^{-1/2} du$ $= \int_1^4 (u^{1/2} - u^{-1/2}) du$ $= \left[\frac{2}{3} u^{3/2} - 2u^{1/2} \right]_1^4$ $= (16/3 - 4) - (2/3 - 2)$ $= 2\frac{2}{3}$</p> <p>OR Let $u = x, v' = (1+x)^{-1/2}$ $\Rightarrow u' = 1, v = 2(1+x)^{1/2}$ \Rightarrow $\int_0^3 x(1+x)^{-1/2} dx = \left[2x(1+x)^{1/2} \right]_0^3 - \int_0^3 2(1+x)^{1/2} dx$ $= \left[2x(1+x)^{1/2} - \frac{4}{3}(1+x)^{3/2} \right]_0^3$ $= (2 \times 3 \times 2 - 4 \times 8/3) - (0 - 4/3)$ $= 2\frac{2}{3}$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1dep</p> <p>A1cao</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>A1cao</p> <p>[5]</p>	<p>$\int (u-1)u^{-1/2} (du)$ *</p> <p>$\int (u^{1/2} - u^{-1/2})(du)$</p> <p>$\left[\frac{2}{3} u^{3/2} - 2u^{1/2} \right]$ o.e.</p> <p>upper-lower dep 1st M1 and integration</p> <p>or 2.6 but must be exact</p> <p>ignore limits, condone no dx</p> <p>ignore limits</p> <p>or 2.6 but must be exact</p>	<p>condone no du, missing bracket, ignore limits</p> <p>e.g. $\left[\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2} \right]$; ignore limits</p> <p>with correct limits e.g. 1, 4 for u or 0, 3 for x or using $w = (1+x)^{1/2} \Rightarrow$ $\int \frac{(w^2-1)2w}{w} (dw)$ M1 $= \int 2(w^2-1)(dw)$ A1 = $\left[\frac{2}{3} w^3 - 2w \right]$ A1</p> <p>upper-lower with correct limits ($w = 1, 2$) M1</p> <p>8/3 A1 cao</p> <p>*If $\int_1^4 (u-1)u^{-1/2} du$ done by parts: $2u^{1/2}(u-1) - \int 2u^{1/2} du$ A1 $[2u^{1/2}(u-1) - 4u^{3/2}/3]$ A1 substituting correct limits M1 8/3 A1cao</p>

Question		Answer	Marks	Guidance
7	(i)	$3^5 + 2 = 245$ [which is not prime]	M1 A1 [2]	Attempt to find counter-example correct counter-example identified If A0, allow M1 for $3^n + 2$ correctly evaluated for 3 values of n
7	(ii)	$(3^0 = 1), 3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81, \dots$ so units digits cycle through 1, 3, 9, 7, 1, 3, ... so cannot be a '5'. OR 3^n is not divisible by 5 all numbers ending in '5' are divisible by 5. so its last digit cannot be a '5'	M1 A1 B1 B1 [2]	Evaluate 3^n for $n = 0$ to 4 or 1 to 5 allow just final digit written must state conclusion for B2
8	(i)	translation in the x -direction of $\pi/4$ to the right translation in y -direction of 1 unit up.	M1 A1 M1 A1 [4]	allow 'shift', 'move' oe (eg using vector) allow 'shift', 'move' oe (eg using vector) If just vectors given withhold one 'A' mark only 'Translate $\begin{pmatrix} \pi/4 \\ 1 \end{pmatrix}$ ' is 4 marks; if this is followed by an additional incorrect transformation, SC M1M1A1A0 $\begin{pmatrix} \pi/4 \\ 1 \end{pmatrix}$ only is M2A1A0

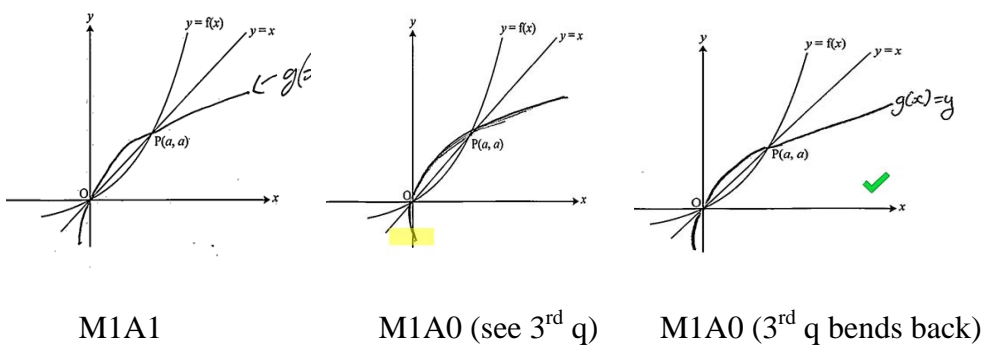
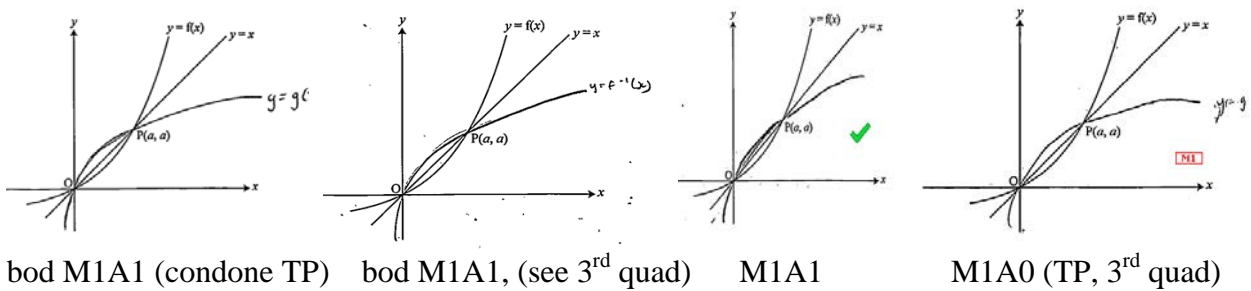
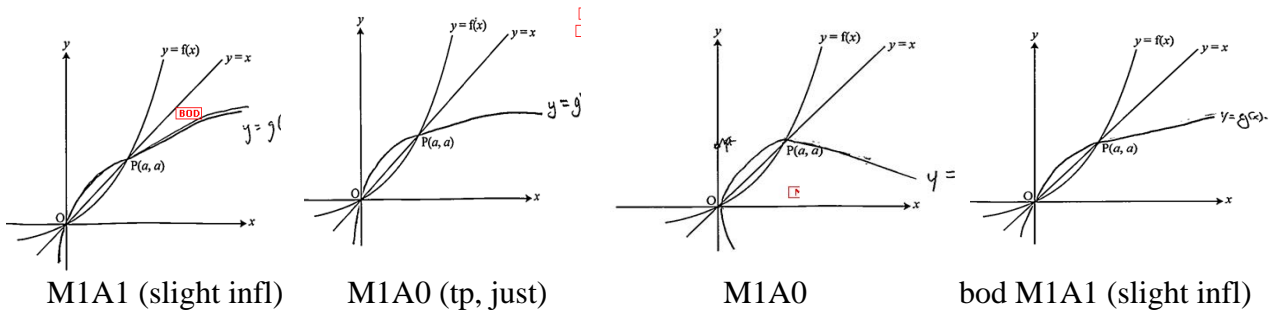
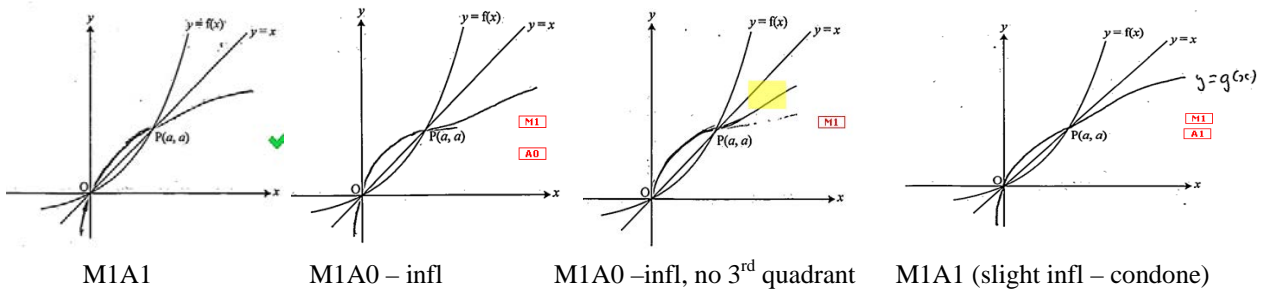
Question	Answer	Marks		Guidance
8 (ii)	$g(x) = \frac{2 \sin x}{\sin x + \cos x}$ $g'(x) = \frac{(\sin x + \cos x)2 \cos x - 2 \sin x(\cos x - \sin x)}{(\sin x + \cos x)^2}$ $= \frac{2 \sin x \cos x + 2 \cos^2 x - 2 \sin x \cos x + 2 \sin^2 x}{(\sin x + \cos x)^2}$ $= \frac{2 \cos^2 x + 2 \sin^2 x}{(\sin x + \cos x)^2} = \frac{2(\cos^2 x + \sin^2 x)}{(\sin x + \cos x)^2}$ $= \frac{2}{(\sin x + \cos x)^2} *$ <p>When $x = \pi/4$, $g'(\pi/4) = 2/(1/\sqrt{2} + 1/\sqrt{2})^2$</p> $= 1$ <p>$f'(x) = \sec^2 x$ $f'(0) = \sec^2(0) = 1$, [so gradient the same here]</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[7]</p>	<p>Quotient (or product) rule consistent with their deriv</p> <p>Correct expanded expression (could leave the '2' as a factor)</p> <p>NB AG</p> <p>substituting $\pi/4$ into correct deriv</p> <p>o.e., e.g. $1/\cos^2 x$</p>	<p>(Can deal with num and denom separately)</p> <p>$\frac{vu' - uv'}{v^2}$; allow one slip, missing brackets</p> <p>$\frac{uv' - vu'}{v^2}$ is M0. Condone $\cos x^2$, $\sin x^2$</p> <p>must take out 2 as a factor or state $\sin^2 x + \cos^2 x = 1$</p>

Question	Answer	Marks		Guidance
8 (iii)	$\int_0^{\pi/4} f(x) dx = \int_0^{\pi/4} \frac{\sin x}{\cos x} dx$ <p>let $u = \cos x$, $du = -\sin x dx$ when $x = 0$, $u = 1$, when $x = \pi/4$, $u = 1/\sqrt{2}$</p> $= \int_1^{1/\sqrt{2}} -\frac{1}{u} du$ $= \int_{1/\sqrt{2}}^1 \frac{1}{u} du^*$ $= [\ln u]_{1/\sqrt{2}}^1$ $= \ln 1 - \ln(1/\sqrt{2})$ $= \ln \sqrt{2} = \ln 2^{1/2} = \frac{1}{2} \ln 2$	M1 A1 M1 A1 [4]	substituting to get $\int -1/u (du)$ NB AG $[\ln u]$ $\ln \sqrt{2}$, $\frac{1}{2} \ln 2$ or $-\ln(1/\sqrt{2})$	ignore limits here, condone no du but not dx allow $\int 1/u \cdot -du$ but for A1 must deal correctly with the $-ve$ sign by interchanging limits mark final answer
8 (iv)	Area = area in part (iii) translated up 1 unit. So = $\frac{1}{2} \ln 2 + 1 \times \pi/4 = \frac{1}{2} \ln 2 + \pi/4$.	M1 A1cao [2]	soi from $\pi/4$ added oe (as above)	or $\int_{\pi/4}^{\pi/2} (1 + \tan(x - \pi/4)) dx = [x + \ln \sec(x - \pi/4)]_{\pi/4}^{\pi/2}$ $= \pi/2 + \ln \sqrt{2} - \pi/4 = \pi/4 + \ln \sqrt{2}$ B2
9 (i)	At $P(a, a)$ $g(a) = a$ so $\frac{1}{2}(e^a - 1) = a$ $\Rightarrow e^a = 1 + 2a^*$	B1 [1]	NB AG	
9 (ii)	$A = \int_0^a \frac{1}{2}(e^x - 1) dx$ $= \frac{1}{2} [e^x - x]_0^a$ $= \frac{1}{2} (e^a - a - e^0)$ $= \frac{1}{2} (1 + 2a - a - 1) = \frac{1}{2} a^*$ area of triangle = $\frac{1}{2} a^2$ area between curve and line = $\frac{1}{2} a^2 - \frac{1}{2} a$	M1 B1 A1 A1 B1 B1cao [6]	correct integral and limits integral of $e^x - 1$ is $e^x - x$ NB AG mark final answer	limits can be implied from subsequent work

Question	Answer	Marks	Guidance
9 (iii)	$y = \frac{1}{2}(e^x - 1)$ swap x and y $x = \frac{1}{2}(e^y - 1)$ $\Rightarrow 2x = e^y - 1$ $\Rightarrow 2x + 1 = e^y$ $\Rightarrow \ln(2x + 1) = y^*$ $\Rightarrow g(x) = \ln(2x + 1)$ Sketch: recognisable attempt to reflect in $y = x$ Good shape	M1 A1 A1 M1 A1 [5]	Attempt to invert – one valid step $y = \ln(2x + 1)$ or $g(x) = \ln(2x + 1)$ AG through O and (a, a) no obvious inflexion or TP, extends to third quadrant, without gradient becoming too negative
9 (iv)	$f'(x) = \frac{1}{2}e^x$ $g'(x) = \frac{2}{2x + 1}$ $g'(a) = \frac{2}{2a + 1}$, $f'(a) = \frac{1}{2}e^a$ so $g'(a) = \frac{2}{e^a}$ or $f'(a) = \frac{1}{2}(2a + 1)$ $= \frac{1}{\frac{1}{2}e^a}$ $= (2a + 1)/2$ $[= 1/f'(a)]$ $[= 1/g'(a)]$ tangents are reflections in $y = x$	B1 M1 A1 B1 M1 A1 B1 [7]	$\frac{1}{2x + 1}$ (or $1/u$ with $u = 2x + 1$) ... $\dots \times 2$ to get $\frac{2}{2x + 1}$ either $g'(a)$ or $f'(a)$ correct soi substituting $e^a = 1 + 2a$ establishing $f'(a) = 1/g'(a)$ must mention tangents
			merely swapping x and y is not 'one step' apply a similar scheme if they start with $g(x)$ and invert to get $f(x)$. or $g f(x) = g((e^x - 1)/2)$ M1 $= \ln(1 + e^x - 1) = \ln(e^x)$ A1 = x A1 similar scheme for fg See appendix for examples

APPENDIX 1

Exemplar marking of 9(iii)



OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning

Telephone: 01223 553998

Facsimile: 01223 552627

Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553

© OCR 2013

