

**Wednesday 23 January 2013 – Morning**

**AS GCE MATHEMATICS**

**4725/01** Further Pure Mathematics 1

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4725/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

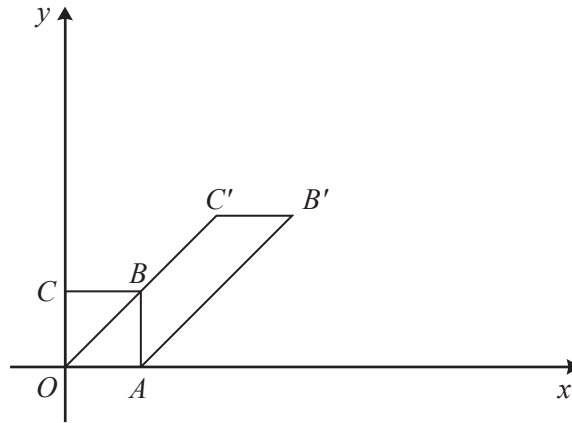
- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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- 1 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} a & 1 \\ 1 & 4 \end{pmatrix}$ , where  $a \neq \frac{1}{4}$ , and  $\mathbf{I}$  denotes the  $2 \times 2$  identity matrix. Find
- (i)  $2\mathbf{A} - 3\mathbf{I}$ , [3]
- (ii)  $\mathbf{A}^{-1}$ . [2]
- 2 Find  $\sum_{r=1}^n (r-1)(r+1)$ , giving your answer in a fully factorised form. [6]
- 3 The complex number  $2 - i$  is denoted by  $z$ .
- (i) Find  $|z|$  and  $\arg z$ . [2]
- (ii) Given that  $az + bz^* = 4 - 8i$ , find the values of the real constants  $a$  and  $b$ . [5]
- 4 The quadratic equation  $x^2 + x + k = 0$  has roots  $\alpha$  and  $\beta$ .
- (i) Use the substitution  $x = 2u + 1$  to obtain a quadratic equation in  $u$ . [2]
- (ii) Hence, or otherwise, find the value of  $\left(\frac{\alpha-1}{2}\right)\left(\frac{\beta-1}{2}\right)$  in terms of  $k$ . [2]
- 5 By using the determinant of an appropriate matrix, find the values of  $\lambda$  for which the simultaneous equations
- $$\begin{aligned} 3x + 2y + 4z &= 5, \\ \lambda y + z &= 1, \\ x + \lambda y + \lambda z &= 4, \end{aligned}$$
- do not have a unique solution for  $x$ ,  $y$  and  $z$ . [6]

6



The diagram shows the unit square  $OABC$ , and its image  $OAB'C'$  after a transformation. The points have the following coordinates:  $A(1, 0)$ ,  $B(1, 1)$ ,  $C(0, 1)$ ,  $B'(3, 2)$  and  $C'(2, 2)$ .

(i) Write down the matrix,  $\mathbf{X}$ , for this transformation. [2]

(ii) The transformation represented by  $\mathbf{X}$  is equivalent to a transformation P followed by a transformation Q. Give geometrical descriptions of a pair of possible transformations P and Q and state the matrices that represent them. [6]

(iii) Find the matrix that represents transformation Q followed by transformation P. [2]

7 (i) Sketch on a single Argand diagram the loci given by

(a)  $|z| = 2$ , [2]

(b)  $\arg(z - 3 - i) = \pi$ . [3]

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z| \leq 2 \text{ and } 0 \leq \arg(z - 3 - i) \leq \pi. \quad [2]$$

8 (i) Show that  $\frac{1}{r} - \frac{3}{r+1} + \frac{2}{r+2} \equiv \frac{2-r}{r(r+1)(r+2)}$ . [2]

(ii) Hence show that  $\sum_{r=1}^n \frac{2-r}{r(r+1)(r+2)} = \frac{n}{(n+1)(n+2)}$ . [5]

(iii) Find the value of  $\sum_{r=2}^{\infty} \frac{2-r}{r(r+1)(r+2)}$ . [2]

- 9 (i) Show that  $(\alpha\beta + \beta\gamma + \gamma\alpha)^2 \equiv \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$ . [3]
- (ii) It is given that  $\alpha$ ,  $\beta$  and  $\gamma$  are the roots of the cubic equation  $x^3 + px^2 - 4x + 3 = 0$ , where  $p$  is a constant. Find the value of  $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$  in terms of  $p$ . [5]
- 10 The sequence  $u_1, u_2, u_3, \dots$  is defined by  $u_1 = 2$  and  $u_{n+1} = \frac{u_n}{1 + u_n}$  for  $n \geq 1$ .
- (i) Find  $u_2$  and  $u_3$ , and show that  $u_4 = \frac{2}{7}$ . [3]
- (ii) Hence suggest an expression for  $u_n$ . [2]
- (iii) Use induction to prove that your answer to part (ii) is correct. [5]

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