OCR RECOGNISING ACHIEVEMENT

Wednesday 23 January 2013 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 1 \\ 1 & 4 \end{pmatrix}$, where $a \neq \frac{1}{4}$, and **I** denotes the 2×2 identity matrix. Find

(i) $2\mathbf{A} - 3\mathbf{I}$, **[3]**

(ii) $\mathbf{A}^{-1}$. **[2]**

2 Find $\sum_{r=1}^{n} (r-1)(r+1)$, giving your answer in a fully factorised form. **[6]**

3 The complex number $2 - \mathrm{i}$ is denoted by z.

(i) Find $|z|$ and $\arg z$. **[2]**

(ii) Given that $az + bz^* = 4 - 8\mathrm{i}$, find the values of the real constants a and b. **[5]**

4 The quadratic equation $x^2 + x + k = 0$ has roots α and β.

(i) Use the substitution $x = 2u + 1$ to obtain a quadratic equation in u. **[2]**

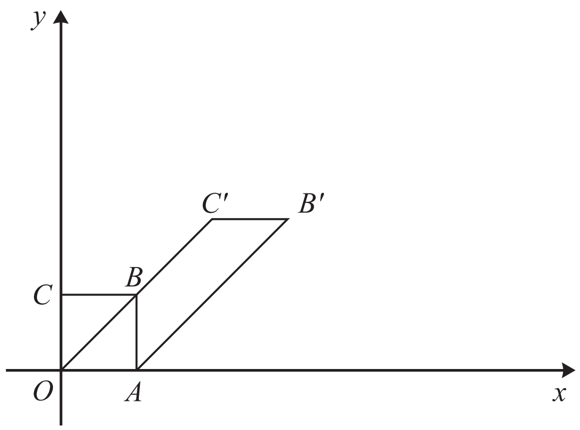
(ii) Hence, or otherwise, find the value of $\left(\frac{\alpha - 1}{2}\right)\left(\frac{\beta - 1}{2}\right)$ in terms of k. **[2]**

5 By using the determinant of an appropriate matrix, find the values of λ for which the simultaneous equations

$$\begin{aligned} 3x + 2y + 4z &= 5, \\ \lambda y + z &= 1, \\ x + \lambda y + \lambda z &= 4, \end{aligned}$$

do not have a unique solution for x, y and z. **[6]**

6



The diagram shows the unit square $OABC$, and its image $OAB'C'$ after a transformation. The points have the following coordinates: $A(1, 0)$, $B(1, 1)$, $C(0, 1)$, $B'(3, 2)$ and $C'(2, 2)$.

(i) Write down the matrix, $\mathbf{X}$, for this transformation. **[2]**

(ii) The transformation represented by $\mathbf{X}$ is equivalent to a transformation P followed by a transformation Q. Give geometrical descriptions of a pair of possible transformations P and Q and state the matrices that represent them. **[6]**

(iii) Find the matrix that represents transformation Q followed by transformation P. **[2]**

7 **(i)** Sketch on a single Argand diagram the loci given by

(a) $|z| = 2$, **[2]**

(b) $\arg(z - 3 - \mathrm{i}) = \pi$. **[3]**

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z| \leqslant 2 \text{ and } 0 \leqslant \arg(z - 3 - \mathrm{i}) \leqslant \pi.$$ **[2]**

8 **(i)** Show that $\dfrac{1}{r} - \dfrac{3}{r+1} + \dfrac{2}{r+2} \equiv \dfrac{2-r}{r(r+1)(r+2)}$. **[2]**

(ii) Hence show that $\displaystyle\sum_{r=1}^{n} \frac{2-r}{r(r+1)(r+2)} = \frac{n}{(n+1)(n+2)}$. **[5]**

(iii) Find the value of $\displaystyle\sum_{r=2}^{\infty} \frac{2-r}{r(r+1)(r+2)}$. **[2]**

9 **(i)** Show that $(\alpha\beta + \beta\gamma + \gamma\alpha)^2 \equiv \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$. **[3]**

(ii) It is given that α, β and γ are the roots of the cubic equation $x^3 + px^2 - 4x + 3 = 0$, where p is a constant. Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$ in terms of p. **[5]**

10 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 2$ and $u_{n+1} = \dfrac{u_n}{1 + u_n}$ for $n \geqslant 1$.

(i) Find u_2 and u_3, and show that $u_4 = \frac{2}{7}$. **[3]**

(ii) Hence suggest an expression for u_n. **[2]**

(iii) Use induction to prove that your answer to part **(ii)** is correct. **[5]**