OCR
RECOGNISING ACHIEVEMENT

Monday 14 January 2013 – Morning

A2 GCE MATHEMATICS

4726/01 Further Pure Mathematics 2

QUESTION PAPER

4733750113

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4726/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Express $\dfrac{5x}{(x-1)(x^2+4)}$ in partial fractions. **[5]**

2 The equation of a curve is $y = \dfrac{x^2-3}{x-1}$.

(i) Find the equations of the asymptotes of the curve. **[3]**

(ii) Write down the coordinates of the points where the curve cuts the axes. **[1]**

(iii) Show that the curve has no stationary points. **[3]**

(iv) Sketch the curve and the asymptotes. **[3]**

3 By first expressing $\cosh x$ and $\sinh x$ in terms of exponentials, solve the equation

$$3\cosh x - 4\sinh x = 7,$$

giving your answer in an exact logarithmic form. **[6]**

4 You are given that $I_n = \displaystyle\int_0^1 x^n \mathrm{e}^{2x}\,\mathrm{d}x$ for $n \geqslant 0$.

(i) Show that $I_n = \frac{1}{2}\mathrm{e}^2 - \frac{1}{2}nI_{n-1}$ for $n \geqslant 1$. **[4]**

(ii) Find I_3 in terms of e. **[4]**

5 You are given that $\mathrm{f}(x) = \mathrm{e}^{-x}\sin x$.

(i) Find $\mathrm{f}(0)$ and $\mathrm{f}'(0)$. **[3]**

(ii) Show that $\mathrm{f}''(x) = -2\,\mathrm{f}'(x) - 2\,\mathrm{f}(x)$ and hence, or otherwise, find $\mathrm{f}''(0)$. **[4]**

(iii) Find a similar expression for $\mathrm{f}'''(x)$ and hence, or otherwise, find $\mathrm{f}'''(0)$. **[2]**

(iv) Find the Maclaurin series for $\mathrm{f}(x)$ up to and including the term in x^3. **[2]**

6 By first completing the square, find $\int_0^1 \frac{1}{\sqrt{x^2+4x+8}}\,\mathrm{d}x$, giving your answer in an exact logarithmic form. **[6]**

7 A curve has polar equation $r = 5\sin 2\theta$ for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(i) Sketch the curve, indicating the line of symmetry and stating the polar coordinates of the point P on the curve which is furthest away from the pole. **[4]**

(ii) Calculate the area enclosed by the curve. **[3]**

(iii) Find the cartesian equation of the tangent to the curve at P. **[3]**

(iv) Show that a cartesian equation of the curve is $\left(x^2+y^2\right)^3 = \left(10xy\right)^2$. **[3]**

8 It is required to solve the equation $\ln(x-1) - x + 3 = 0$.

You are given that there are two roots, α and β, where $1.1 < \alpha < 1.2$ and $4.1 < \beta < 4.2$.

(i) The root β can be found using the iterative formula

$$x_{n+1} = \ln(x_n - 1) + 3.$$

(a) Using this iterative formula with $x_1 = 4.15$, find β correct to 3 decimal places. Show all your working. **[2]**

(b) Explain with the aid of a sketch why this iterative formula will not converge to α whatever initial value is taken. **[3]**

(ii) (a) Show that the Newton-Raphson iterative formula for this equation can be written in the form

$$x_{n+1} = \frac{3 - 2x_n - (x_n - 1)\ln(x_n - 1)}{2 - x_n}.$$ **[5]**

(b) Use this formula with $x_1 = 1.2$ to find α correct to 3 decimal places. **[3]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.