OCR
RECOGNISING ACHIEVEMENT

Wednesday 30 January 2013 – Morning

A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4727
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Two planes have equations

$$x + 2y + 5z = 12 \quad \text{and} \quad 2x - y + 3z = 5.$$

(i) Find the acute angle between the planes. **[3]**

(ii) Find a vector equation of the line of intersection of the planes. **[4]**

2 The elements of a group G are the complex numbers $a + b\mathrm{i}$ where $a, b \in \{0, 1, 2, 3, 4\}$. These elements are combined under the operation of addition modulo 5.

(i) State the identity element and the order of G. **[2]**

(ii) Write down the inverse of $2 + 4\mathrm{i}$. **[1]**

(iii) Show that every non-zero element of G has order 5. **[3]**

3 Solve the differential equation $x\dfrac{\mathrm{d}y}{\mathrm{d}x} - 3y = x^4\mathrm{e}^{2x}$ for y in terms of x, given that $y = 0$ when $x = 1$. **[8]**

4 The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix}$$

respectively.

(i) Find the shortest distance between the lines. **[5]**

(ii) Find a cartesian equation of the plane which contains l_1 and which is parallel to l_2. **[2]**

5 **(i)** Solve the equation $z^5 = 1$, giving your answers in polar form. **[2]**

(ii) Hence, by considering the equation $(z + 1)^5 = z^5$, show that the roots of

$$5z^4 + 10z^3 + 10z^2 + 5z + 1 = 0$$

can be expressed in the form $\dfrac{1}{\mathrm{e}^{\mathrm{i}\theta} - 1}$, stating the values of θ. **[5]**

6 The differential equation $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4y = \sin kx$ is to be solved, where k is a constant.

(i) In the case $k = 2$, by using a particular integral of the form $ax\cos 2x + bx\sin 2x$, find the general solution. **[7]**

(ii) Describe briefly the behaviour of y when $x \to \infty$. **[2]**

(iii) In the case $k \neq 2$, explain whether y would exhibit the same behaviour as in part **(ii)** when $x \to \infty$. **[2]**

7 Let $S = \mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{2\mathrm{i}\theta} + \mathrm{e}^{3\mathrm{i}\theta} + \ldots + \mathrm{e}^{10\mathrm{i}\theta}$.

(i) (a) Show that, for $\theta \neq 2n\pi$, where n is an integer,

$$S = \frac{\mathrm{e}^{\frac{1}{2}\mathrm{i}\theta}\left(\mathrm{e}^{10\mathrm{i}\theta} - 1\right)}{2\mathrm{i}\sin\left(\frac{1}{2}\theta\right)}.$$ **[4]**

(b) State the value of S for $\theta = 2n\pi$, where n is an integer. **[1]**

(ii) Hence show that, for $\theta \neq 2n\pi$, where n is an integer,

$$\cos\theta + \cos 2\theta + \cos 3\theta + \ldots + \cos 10\theta = \frac{\sin\left(\frac{21}{2}\theta\right)}{2\sin\left(\frac{1}{2}\theta\right)} - \frac{1}{2}.$$ **[3]**

(iii) Hence show that $\theta = \frac{1}{11}\pi$ is a root of $\cos\theta + \cos 2\theta + \cos 3\theta + \ldots + \cos 10\theta = 0$ and find another root in the interval $0 < \theta < \frac{1}{4}\pi$. **[4]**

8 A multiplicative group H has the elements $\{e, a, a^2, a^3, w, aw, a^2w, a^3w\}$ where e is the identity, elements a and w have orders 4 and 2 respectively and $wa = a^3w$.

(i) Show that $wa^2 = a^2w$ and also that $wa^3 = aw$. **[6]**

(ii) Hence show that each of aw, a^2w and a^3w has order 2. **[4]**

(iii) Find two non-cyclic subgroups of H of order 4, and show that they are not cyclic. **[4]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.