

**Wednesday 30 January 2013 – Morning**

**A2 GCE MATHEMATICS**

**4727/01 Further Pure Mathematics 3**

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4727
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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1 Two planes have equations

$$x + 2y + 5z = 12 \quad \text{and} \quad 2x - y + 3z = 5.$$

(i) Find the acute angle between the planes. [3]

(ii) Find a vector equation of the line of intersection of the planes. [4]

2 The elements of a group  $G$  are the complex numbers  $a + bi$  where  $a, b \in \{0, 1, 2, 3, 4\}$ . These elements are combined under the operation of addition modulo 5.

(i) State the identity element and the order of  $G$ . [2]

(ii) Write down the inverse of  $2 + 4i$ . [1]

(iii) Show that every non-zero element of  $G$  has order 5. [3]

3 Solve the differential equation  $x \frac{dy}{dx} - 3y = x^4 e^{2x}$  for  $y$  in terms of  $x$ , given that  $y = 0$  when  $x = 1$ . [8]

4 The lines  $l_1$  and  $l_2$  have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix}$$

respectively.

(i) Find the shortest distance between the lines. [5]

(ii) Find a cartesian equation of the plane which contains  $l_1$  and which is parallel to  $l_2$ . [2]

5 (i) Solve the equation  $z^5 = 1$ , giving your answers in polar form. [2]

(ii) Hence, by considering the equation  $(z + 1)^5 = z^5$ , show that the roots of

$$5z^4 + 10z^3 + 10z^2 + 5z + 1 = 0$$

can be expressed in the form  $\frac{1}{e^{i\theta} - 1}$ , stating the values of  $\theta$ . [5]

6 The differential equation  $\frac{d^2y}{dx^2} + 4y = \sin kx$  is to be solved, where  $k$  is a constant.

(i) In the case  $k = 2$ , by using a particular integral of the form  $ax \cos 2x + bx \sin 2x$ , find the general solution. [7]

(ii) Describe briefly the behaviour of  $y$  when  $x \rightarrow \infty$ . [2]

(iii) In the case  $k \neq 2$ , explain whether  $y$  would exhibit the same behaviour as in part (ii) when  $x \rightarrow \infty$ . [2]

7 Let  $S = e^{i\theta} + e^{2i\theta} + e^{3i\theta} + \dots + e^{10i\theta}$ .

(i) (a) Show that, for  $\theta \neq 2n\pi$ , where  $n$  is an integer,

$$S = \frac{e^{\frac{1}{2}i\theta}(e^{10i\theta} - 1)}{2i \sin\left(\frac{1}{2}\theta\right)}. \quad [4]$$

(b) State the value of  $S$  for  $\theta = 2n\pi$ , where  $n$  is an integer. [1]

(ii) Hence show that, for  $\theta \neq 2n\pi$ , where  $n$  is an integer,

$$\cos \theta + \cos 2\theta + \cos 3\theta + \dots + \cos 10\theta = \frac{\sin\left(\frac{21}{2}\theta\right)}{2 \sin\left(\frac{1}{2}\theta\right)} - \frac{1}{2}. \quad [3]$$

(iii) Hence show that  $\theta = \frac{1}{11}\pi$  is a root of  $\cos \theta + \cos 2\theta + \cos 3\theta + \dots + \cos 10\theta = 0$  and find another root in the interval  $0 < \theta < \frac{1}{4}\pi$ . [4]

8 A multiplicative group  $H$  has the elements  $\{e, a, a^2, a^3, w, aw, a^2w, a^3w\}$  where  $e$  is the identity, elements  $a$  and  $w$  have orders 4 and 2 respectively and  $wa = a^3w$ .

(i) Show that  $wa^2 = a^2w$  and also that  $wa^3 = aw$ . [6]

(ii) Hence show that each of  $aw$ ,  $a^2w$  and  $a^3w$  has order 2. [4]

(iii) Find two non-cyclic subgroups of  $H$  of order 4, and show that they are not cyclic. [4]

**THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.**



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