

**Friday 25 January 2013 – Afternoon**

**A2 GCE MATHEMATICS**

**4730/01** Mechanics 3

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4730/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION FOR CANDIDATES**

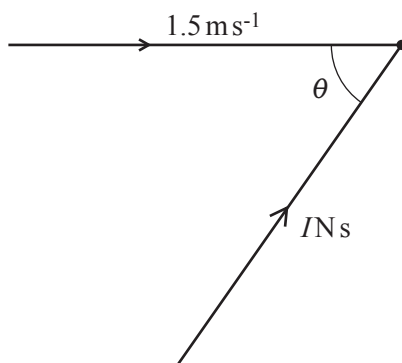
This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

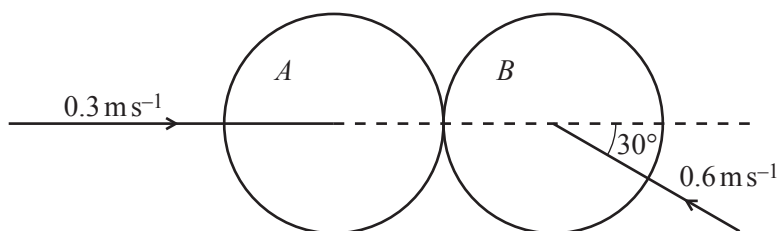
- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1



A ball of mass  $0.6 \text{ kg}$  is moving with speed  $1.5 \text{ m s}^{-1}$  in a straight line. It is struck by an impulse  $I \text{ N s}$  acting at an acute angle  $\theta$  to its direction of motion (see diagram). The impulse causes the direction of motion of the ball to change by an acute angle  $\alpha$ , where  $\sin \alpha = \frac{8}{17}$ . After the impulse acts the ball is moving with a speed of  $3.4 \text{ m s}^{-1}$ . Find  $I$  and  $\theta$ . [5]

- 2 Two uniform smooth spheres  $A$  and  $B$ , of equal radius and equal mass, are moving towards each other on a horizontal surface. Immediately before they collide,  $A$  has speed  $0.3 \text{ m s}^{-1}$  along the line of centres and  $B$  has speed  $0.6 \text{ m s}^{-1}$  at an angle of  $30^\circ$  to the line of centres (see diagram).



After the collision, the direction of motion of  $B$  is at right angles to its original direction of motion. Find

- (i) the speed of  $B$  after the collision, [3]
  - (ii) the speed and direction of motion of  $A$  after the collision, [3]
  - (iii) the coefficient of restitution between  $A$  and  $B$ . [3]
- 3 At time  $t = 0 \text{ s}$  a particle  $P$ , of mass  $0.3 \text{ kg}$ , is  $1 \text{ m}$  away from a point  $O$  on a smooth horizontal plane and is moving away from  $O$  with speed  $\sqrt{5} \text{ m s}^{-1}$ . The only horizontal force acting on  $P$  has magnitude  $1.5x \text{ N}$ , where  $x$  is the distance  $OP$ , and acts away from  $O$ .
- (i) Show that the speed of  $P$ ,  $v \text{ m s}^{-1}$ , is given by  $v = \sqrt{5}x$ . [4]
  - (ii) Find an expression for  $v$  in terms of  $t$ . [4]

- 4 A smooth cylinder of radius  $a$  m is fixed with its axis horizontal and  $O$  is the centre of a cross-section. Particle  $P$ , of mass  $0.4$  kg, and particle  $Q$ , of mass  $0.6$  kg, are connected by a light inextensible string of length  $\pi a$  m. The string is held at rest with  $P$  and  $Q$  at opposite ends of the horizontal diameter of the cross-section through  $O$  (see Fig. 1). The string is released and  $Q$  begins to descend. When  $OP$  has rotated through  $\theta$  radians, with  $P$  remaining in contact with the cylinder, the speed of each particle is  $v$  m s<sup>-1</sup> (see Fig. 2).

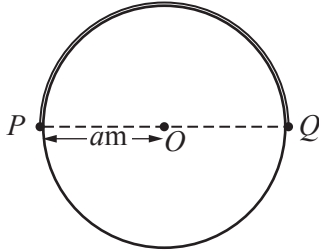


Fig. 1

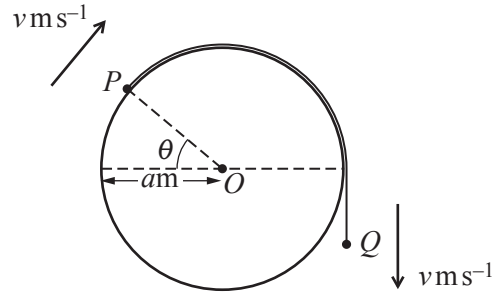


Fig. 2

- (i) Show that  $v^2 = 3.92a(3\theta - 2\sin\theta)$  and find an expression in terms of  $\theta$  for the normal force of the cylinder on  $P$  at this time. [9]
- (ii) Given that  $P$  leaves the surface of the cylinder when  $\theta = \alpha$ , show that  $\sin\alpha = k\alpha$  where  $k$  is a constant to be found. [2]
- 5 A particle  $P$ , of mass  $2.5$  kg, is in equilibrium suspended from a fixed point  $A$  by a light elastic string of natural length  $3$  m and modulus of elasticity  $36.75$  N. Another particle  $Q$ , of mass  $1$  kg, is released from rest at  $A$  and falls freely until it reaches  $P$  and becomes attached to it.

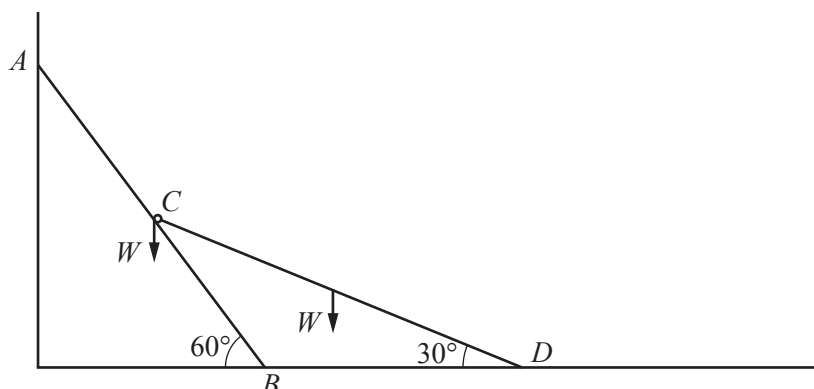
- (i) Show that the speed of the combined particles, immediately after  $Q$  becomes attached to  $P$ , is  $2\sqrt{2}$  m s<sup>-1</sup>. [6]

The combined particles fall a further distance  $X$  m before coming to instantaneous rest.

- (ii) Find a quadratic equation satisfied by  $X$ , and show that it simplifies to  $35X^2 - 56X - 80 = 0$ . [6]

[Questions 6 and 7 are printed overleaf]

- 6 A uniform rod  $AB$ , of weight  $W$  and length  $2l$  is in equilibrium at  $60^\circ$  to the horizontal with  $A$  resting against a smooth vertical plane and  $B$  resting on a rough section of a horizontal plane. Another uniform rod  $CD$ , of length  $\sqrt{3}l$  and weight  $W$ , is freely jointed to the mid-point of  $AB$  at  $C$ ; its other end  $D$  rests on a smooth section of the horizontal plane.  $CD$  is inclined at  $30^\circ$  to the horizontal (see diagram).



- (i) Show that the force exerted by the horizontal plane on  $CD$  is  $\frac{1}{2}W$ . Find the normal component of the force exerted by the horizontal plane on  $AB$ . [5]
- (ii) Find the magnitude and direction of the force exerted by  $CD$  on  $AB$ . [3]
- (iii) Given that  $AB$  is in limiting equilibrium, find the coefficient of friction between  $AB$  and the horizontal plane. [5]
- 7 A simple pendulum consists of a light inextensible string of length  $0.8\text{ m}$  and a particle  $P$  of mass  $m\text{ kg}$ . The pendulum is hanging vertically at rest from a fixed point  $O$  when  $P$  is given a horizontal velocity of  $0.3\text{ ms}^{-1}$ .
- (i) Show that, in the subsequent motion, the maximum angle between the string and the downward vertical is  $0.107$  radians, correct to 3 significant figures. [3]
- (ii) Show that the motion may be modelled as simple harmonic motion, and find the period of this motion. [5]
- (iii) Find the time after the start of the motion when the velocity of the particle is first  $-0.2\text{ ms}^{-1}$  and find the angular displacement of  $OP$  from the downward vertical at this time. [6]

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