

**Friday 18 January 2013 – Afternoon**

**A2 GCE MATHEMATICS**

**4733/01** Probability & Statistics 2

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4733/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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- 1 A random variable has the distribution  $B(n, p)$ . It is required to test  $H_0: p = \frac{2}{3}$  against  $H_1: p < \frac{2}{3}$  at a significance level as close to 1% as possible, using a sample of size  $n = 8, 9$  or  $10$ . Use tables to find which value of  $n$  gives such a test, stating the critical region for the test and the corresponding significance level. [4]

- 2 A random variable  $C$  has the distribution  $N(\mu, \sigma^2)$ . A random sample of 10 observations of  $C$  is obtained, and the results are summarised as

$$n = 10, \sum c = 380, \sum c^2 = 14602.$$

- (i) Calculate unbiased estimates of  $\mu$  and  $\sigma^2$ . [4]

- (ii) Hence calculate an estimate of the probability that  $C > 40$ . [2]

- 3 A factory produces 9000 music DVDs each day. A random sample of 100 such DVDs is obtained.

- (i) Explain how to obtain this sample using random numbers. [3]

- (ii) Given that 24% of the DVDs produced by the factory are classical, use a suitable approximation to find the probability that, in the sample of 100 DVDs, fewer than 20 are classical. [5]

- 4 A continuous random variable  $X$  has probability density function

$$f(x) = \begin{cases} kx & 0 \leq x \leq a, \\ 0 & \text{otherwise,} \end{cases}$$

where  $k$  and  $a$  are constants.

- (i) State what the letter  $x$  represents. [1]

- (ii) Find  $k$  in terms of  $a$ . [2]

- (iii) Find  $\text{Var}(X)$  in terms of  $a$ . [6]

- 5 In a mine, a deposit of the substance *pitchblende* emits radioactive particles. The number of particles emitted has a Poisson distribution with mean 70 particles per second. The warning level is reached if the total number of particles emitted in one minute is more than 4350.

- (i) A one-minute period is chosen at random. Use a suitable approximation to show that the probability that the warning level is reached during this period is 0.01, correct to 2 decimal places. You should calculate the answer correct to 4 decimal places. [5]

- (ii) Use a suitable approximation to find the probability that in 30 one-minute periods the warning level is reached on at least 4 occasions. (You should use the given rounded value of 0.01 from part (i) in your calculation.) [3]

- 6 Gordon is a cricketer. Over a long period he knows that his population mean score, in number of runs per innings, is 28, and the population standard deviation is 12. In a new season he adopts a different batting style and he finds that in 30 innings using this style his mean score is 28.98.

- (i) Stating a necessary assumption, test at the 5% significance level whether his population mean score has increased. [8]

- (ii) Explain whether it was necessary to use the Central Limit Theorem in part (i). [2]

- 7 The continuous random variable  $X$  has the distribution  $N(\mu, \sigma^2)$ . The mean of a random sample of  $n$  observations of  $X$  is denoted by  $\bar{X}$ . It is given that  $P(\bar{X} < 35.0) = 0.9772$  and  $P(\bar{X} < 20.0) = 0.1587$ .

(i) Obtain a formula for  $\sigma$  in terms of  $n$ . [5]

Two students are discussing this question. Aidan says “If you were told another probability, for instance  $P(\bar{X} > 32) = 0.1$ , you could work out the value of  $\sigma$ .” Binya says, “No, the value of  $P(\bar{X} > 32)$  is fixed by the information you know already.”

(ii) State which of Aidan and Binya is right. If you think that Aidan is right, calculate the value of  $\sigma$  given that  $P(\bar{X} > 32) = 0.1$ . If you think that Binya is right, calculate the value of  $P(\bar{X} > 32)$ . [4]

- 8 In a large city the number of traffic lights that fail in one day of 24 hours is denoted by  $Y$ . It may be assumed that failures occur randomly.

(i) Explain what the statement “failures occur randomly” means. [1]

(ii) State, in context, two different conditions that must be satisfied if  $Y$  is to be modelled by a Poisson distribution, and for each condition explain whether you think it is likely to be met in this context. [4]

(iii) For this part you may assume that  $Y$  is well modelled by the distribution  $Po(\lambda)$ . It is given that  $P(Y = 7) = P(Y = 8)$ . Use an algebraic method to calculate the value of  $\lambda$  and hence calculate the corresponding value of  $P(Y = 7)$ . [5]

- 9 The random variable  $A$  has the distribution  $B(30, p)$ . A test is carried out of the hypotheses  $H_0: p = 0.6$  against  $H_1: p < 0.6$ . The critical region is  $A \leq 13$ .

(i) State the probability that  $H_0$  is rejected when  $p = 0.6$ . [1]

(ii) Find the probability that a Type II error occurs when  $p = 0.5$ . [2]

(iii) It is known that on average  $p = 0.5$  on one day in five, and on other days the value of  $p$  is 0.6. On each day two tests are carried out. If the result of the first test is that  $H_0$  is rejected, the value of  $p$  is adjusted if necessary, to ensure that  $p = 0.6$  for the rest of the day. Otherwise the value of  $p$  remains the same as for the first test. Calculate the probability that the result of the second test is to reject  $H_0$ . [5]

**THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.**



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