

Mark Scheme 4753
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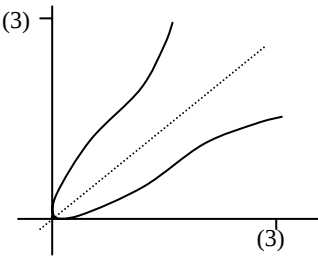
Section A

1 $3x + 2 = 1 \Rightarrow x = -1/3$ $3x + 2 = -1$ $\Rightarrow x = -1$	B1 M1 A1	$x = -1/3$ from a correct method – must be exact
or $(3x + 2)^2 = 1$ $\Rightarrow 9x^2 + 12x + 3 = 0$ $\Rightarrow 3x^2 + 4x + 1 = 0$ $\Rightarrow (3x + 1)(x + 1) = 0$ $\Rightarrow x = -1/3 \text{ or } x = -1$	M1 B1 A1 [3]	Squaring and expanding correctly $x = -1/3$ $x = -1$
2 $x = 1/2$ $\cos \theta = 1/2$ $\Rightarrow \theta = \pi/3$	B1 M1 A1 [3]	M1A0 for 1.04... or 60°
3 $fg(x) = \ln(x^3)$ $= 3 \ln x$ Stretch s.f. 3 in y direction	M1 A1 B1 [3]	$\ln(x^3)$ $= 3 \ln x$
4 $T = 30 + 20e^0 = 50$ $dT/dt = -0.05 \times 20e^{-0.05t} = -e^{-0.05t}$ When $t = 0$, $dT/dt = -1$ When $T = 40$, $40 = 30 + 20e^{-0.05t}$ $\Rightarrow e^{-0.05t} = 1/2$ $\Rightarrow -0.05t = \ln 1/2$ $\Rightarrow t = -20 \ln 1/2 = 13.86.. \text{ (mins)}$	B1 M1 A1cao M1 M1 A1cao [6]	50 correct derivative -1 (or 1) substituting $T = 40$ taking lns correctly or trial and improvement – one value above and one below or 13.9 or 13 mins 52 secs or better www condone secs

5 $\int_0^1 \frac{x}{2x+1} dx$ let $u = 2x + 1$ $\Rightarrow du = 2dx, x = \frac{u-1}{2}$ When $x = 0, u = 1$, when $x = 1, u = 3$ $= \int_1^3 \frac{\frac{1}{2}(u-1)}{u} \frac{1}{2} du = \frac{1}{4} \int_1^3 \frac{u-1}{u} du$ $= \frac{1}{4} \int_1^3 (1 - \frac{1}{u}) du$ $= \frac{1}{4} [u - \ln u]_1^3$ $= \frac{1}{4} [3 - \ln 3 - 1 + \ln 1]$ $= \frac{1}{4} (2 - \ln 3)$	M1 A1 B1 M1 A1 E1 [6]	Substituting $\frac{x}{2x+1} = \frac{u-1}{2u}$ o.e. $\frac{1}{4} \int \frac{u-1}{u} du$ o.e. [condone no du] converting limits dividing through by u $\frac{1}{4} [u - \ln u]$ o.e. – ft their $\frac{1}{4}$ (only) must be some evidence of substitution
6 $y = \frac{x}{2+3\ln x}$ $\Rightarrow \frac{dy}{dx} = \frac{(2+3\ln x) \cdot 1 - x \cdot \frac{3}{x}}{(2+3\ln x)^2}$ $= \frac{2+3\ln x - 3}{(2+3\ln x)^2}$ $= \frac{3\ln x - 1}{(2+3\ln x)^2}$ When $\frac{dy}{dx} = 0, 3\ln x - 1 = 0$ $\Rightarrow \ln x = 1/3$ $\Rightarrow x = e^{1/3}$ $\Rightarrow y = \frac{e^{1/3}}{2+1} = \frac{1}{3} e^{1/3}$	M1 B1 A1 M1 A1cao M1 A1cao [7]	Quotient rule consistent with their derivatives or product rule + chain rule on $(2+3x)^{-1}$ $\frac{d}{dx}(\ln x) = \frac{1}{x}$ soi correct expression their numerator = 0 (or equivalent step from product rule formulation) M0 if denominator = 0 is pursued $x = e^{1/3}$ substituting for their x (correctly) Must be exact: $-0.46\dots$ is M1A0
7 $y^2 + y = x^3 + 2x$ $x = 2 \Rightarrow y^2 + y = 12$ $\Rightarrow y^2 + y - 12 = 0$ $\Rightarrow (y-3)(y+4) = 0$ $\Rightarrow y = 3$ or -4. $2y \frac{dy}{dx} + \frac{dy}{dx} = 3x^2 + 2$ $\Rightarrow \frac{dy}{dx} (2y+1) = 3x^2 + 2$ $\Rightarrow \frac{dy}{dx} = \frac{3x^2 + 2}{2y+1}$ At $(2, 3), \frac{dy}{dx} = \frac{12+2}{6+1} = 2$ At $(2, -4), \frac{dy}{dx} = \frac{12+2}{-8+1} = -2$	M1 A1 A1 M1 A1cao M1 A1 cao A1 cao [8]	Substituting $x = 2$ $y = 3$ $y = -4$ Implicit differentiation – LHS must be correct substituting $x = 2, y = 3$ into their dy/dx, but must require both x and one of their y to be substituted 2 -2

Section B

8 (i) At P, $x \sin 3x = 0$ $\Rightarrow \sin 3x = 0$ $\Rightarrow 3x = \pi$ $\Rightarrow x = \pi/3$	M1 A1 A1cao [3]	$x \sin 3x = 0$ $3x = \pi$ or 180 $x = \pi/3$ or 1.05 or better
(ii) When $x = \pi/6$, $x \sin 3x = \frac{\pi}{6} \sin \frac{\pi}{2} = \frac{\pi}{6}$ $\Rightarrow Q(\pi/6, \pi/6)$ lies on line $y = x$	E1 [1]	$y = \frac{\pi}{6}$ or $x \sin 3x = x \Rightarrow \sin 3x = 1$ etc. Must conclude in radians, and be exact
(iii) $y = x \sin 3x$ $\Rightarrow \frac{dy}{dx} = x \cdot 3 \cos 3x + \sin 3x$ At Q, $\frac{dy}{dx} = \frac{\pi}{6} \cdot 3 \cos \frac{\pi}{2} + \sin \frac{\pi}{2} = 1$ $=$ gradient of $y = x$ So line touches curve at this point	B1 M1 A1cao M1 A1ft E1 [6]	$d/dx (\sin 3x) = 3 \cos 3x$ Product rule consistent with their derivs $3x \cos 3x + \sin 3x$ substituting $x = \pi/6$ into their derivative $= 1$ ft dep 1st M1 $=$ gradient of $y = x$ (www)
(iv) Area under curve $= \int_0^{\pi/6} x \sin 3x dx$ Integrating by parts, $u = x$, $dv/dx = \sin 3x$ $\Rightarrow v = -\frac{1}{3} \cos 3x$ $\int_0^{\pi/6} x \sin 3x dx = \left[-\frac{1}{3} x \cos 3x \right]_0^{\pi/6} + \int_0^{\pi/6} \frac{1}{3} \cos 3x dx$ $= -\frac{1}{3} \cdot \frac{\pi}{6} \cos \frac{\pi}{2} + \frac{1}{3} \cdot 0 \cdot \cos 0 + \left[\frac{1}{9} \sin 3x \right]_0^{\pi/6}$ $= \frac{1}{9}$ Area under line $= \frac{1}{2} \times \frac{\pi}{6} \times \frac{\pi}{6} = \frac{\pi^2}{72}$ So area required $= \frac{\pi^2}{72} - \frac{1}{9}$ $= \frac{\pi^2 - 8}{72}$	M1 A1cao A1ft M1 A1 B1 E1 [7]	Parts with $u = x$ $dv/dx = \sin 3x \Rightarrow$ $v = -\frac{1}{3} \cos 3x$ [condone no negative] $\dots + \left[\frac{1}{9} \sin 3x \right]_0^{\pi/6}$ substituting (correct) limits $\frac{1}{9}$ www $\frac{\pi^2}{72}$ www

9 (i) $f(-x) = \ln[1 + (-x)^2]$ $= \ln[1 + x^2] = f(x)$ Symmetrical about Oy	M1 E1 B1 [3]	If verifies that $f(-x) = f(x)$ using a particular point, allow SCB1 For $f(-x) = \ln(1 + x^2) = f(x)$ allow M1E0 For $f(-x) = \ln(1 + -x^2) = f(x)$ allow M1E0 or 'reflects in Oy', etc
(ii) $y = \ln(1 + x^2)$ let $u = 1 + x^2$ $dy/du = 1/u, du/dx = 2x$ $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ $= \frac{1}{u} \cdot 2x = \frac{2x}{1 + x^2}$ When $x = 2$, $dy/dx = 4/5$.	M1 B1 A1 A1cao [4]	Chain rule 1/u soi
(iii) The function is not one to one for this domain	B1 [1]	Or many to one
(iv)  Domain for $g(x) = 0 \leq x \leq \ln 10$ $y = \ln(1 + x^2) \quad x \leftrightarrow y$ $x = \ln(1 + y^2)$ $\Rightarrow e^x = 1 + y^2$ $\Rightarrow e^x - 1 = y^2$ $\Rightarrow y = \sqrt{e^x - 1}$ so $g(x) = \sqrt{e^x - 1}$ or $g f(x) = g[\ln(1 + x^2)]$ $= \sqrt{e^{\ln(1+x^2)} - 1}$ $= (1 + x^2) - 1$ $= x$	M1 A1 B1 M1 M1 E1 M1 M1 E1 [6]	$g(x)$ is $f(x)$ reflected in $y = x$ Reasonable shape and domain, i.e. no -ve x values, inflection shown, does not cross $y = x$ line Condone y instead of x Attempt to invert function Taking exponentials $g(x) = \sqrt{(e^x - 1)}$ www forming $g f(x)$ or $f g(x)$ $e^{\ln(1+x^2)} = 1 + x^2$ or $\ln(1 + e^x - 1) = x$ www
(v) $g'(x) = \frac{1}{2} (e^x - 1)^{-1/2} \cdot e^x$ $\Rightarrow g'(\ln 5) = \frac{1}{2} (e^{\ln 5} - 1)^{-1/2} \cdot e^{\ln 5}$ $= \frac{1}{2} (5 - 1)^{-1/2} \cdot 5$ $= 5/4$ Reciprocal of gradient at P as tangents are reflections in $y = x$.	B1 B1 M1 E1cao B1 [5]	$\frac{1}{2} u^{-1/2}$ soi $\times e^x$ substituting $\ln 5$ into g' - must be some evidence of substitution Must have idea of reciprocal. Not 'inverse'.