Mark Scheme 4725 June 2005

1.	$6\Sigma r^2 + 2\Sigma r + \Sigma 1$	M1		Consider the sum of three separate terms
	$6\Sigma r^2 = n(n+1)(2n+1)$	A1		Correct formula stated
	$2\Sigma r = n(n+1)$	A1		Correct formula stated
	$\Sigma 1 = n$	A1		Correct term seen
	$n(2n^2 + 4n + 3)$	M1	6	Correct algebraic processes including factorisation and simplification
		A1	6	Obtain given answer correctly
2.	(i) $A^2 = \begin{pmatrix} 3 & 8 \\ 4 & 11 \end{pmatrix}$	M1		Attempt to find A^2 , 2 elements correct
	(411)	A1		All elements correct
	$\mathbf{4A} = \begin{pmatrix} 4 & 8 \\ 4 & 12 \end{pmatrix}$	M1		Use correct matrix 4A
	$\mathbf{A}^2 = 4\mathbf{A} - \mathbf{I}$	A1	4	Obtain given answer correctly
	(ii) $A^{-1} = 4I - A$	M1	2	Multiply answer to (i) by \mathbf{A}^{-1} or obtain \mathbf{A}^{-1} or factorise $\mathbf{A}^2 - 4\mathbf{A}$
		A1	6	Obtain given answer correctly
3.	(i) 22 – 2i	B1B1	2	Correct real and imaginary parts
	(ii) $z^* = 2 - 3i$ 5 - 14i	B1 B1B1	3	Correct conjugate seen or implied Correct real and imaginary parts
	(iii) $\frac{4}{17} + \frac{1}{17}i$	M1 A1	2	Attempt to use <i>w</i> * Obtain correct answer in any form
			7	

4.	$x^2 - y^2 = 21$ and $xy = -10$	M1 A1A1 M1 M1		Attempt to equate real and imaginary parts of $(x + iy)^2$ and 21 –20i Obtain each result Eliminate to obtain a quadratic in x^2 or y^2 Solve to obtain $x = (\pm) 5$ or $y = (\pm) 2$
	±(5-2i)	A1	6	Obtain correct answers as complex numbers
			6	
5.	(i) $\frac{(r+1)^2 - r(r+2)}{(r+2)(r+1)}$	M1		Show correct process for subtracting fractions
	$\frac{1}{(r+1)(r+2)}$	A1	2	Obtain given answer correctly
	(ii) EITHER $\frac{2}{3} - \frac{1}{2} + \frac{3}{4} - \frac{2}{3} \dots \frac{n+1}{n+2} - \frac{n}{n+1}$	M1		Express terms as differences using (i)
	$3 \ 2 \ 4 \ 3 \ n+2 \ n+1$	A1		At least first two and last term correct
	$\frac{n+1}{2}$ _ 1	M1		Show or imply that pairs of terms cancel
	$\frac{1}{n+2}$ $-\frac{1}{2}$	A1	4	Obtain correct answer in any form
	OR	M2		State that $\sum_{r=1}^{n} u_r = f(n+1) - f(1)$
		A1A1		Each term correct
	(iii) $\frac{1}{2}$	B1 ft	1 7	Obtain value from their sum to <i>n</i> terms
			'	
6.	 (i) Circle Centre (0, 2) Radius 2 Straight line Through origin with positive slope 	B1 B1 B1 B1 B1	5	Sketch(s) showing correct features, each mark independent
	(ii) 0 or 0 +0i and 2 + 2i	B1ftB1f t	2	Obtain intersections as complex numbers
8.	(a) (i) $\alpha + \beta = 2$ $\alpha\beta = 4$	B1B1	7 2	Values stated
	(ii) EITHER $\alpha^2 + \beta^2 = -4$	M1 A1	2	Use $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
	OR	M1 A1		Obtain given answer correctly Find numeric values of roots, square and add
	(iii)			Obtain given answer correctly
	$x^2 + 4x + 16 = 0$	B1		State or use $\alpha^2 \beta^2 = 16$

53

	(b) (i) <i>p</i> = 2	M1 A1	3	Or use substitution $u = x^2$ Write down a quadratic equation of correct form or rearrange and square Obtain $x^2 + 4x + 16 = 0$
		M1		Use sum or product of roots to obtain $6p = 12$ Or $6p^3 = 48$ Obtain $p = 2$
	(ii) <i>a</i> = 44	A1	2	Obtain $p = 2$
		M1		Attempt to find $\sum \alpha \beta$ numerically or in terms of <i>p</i> or substitute their 2, 4 or 6 in equation
		A1ft	2	Obtain 11p ²
			11	
9.	(i) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$	B1B1	2	Each column correct
	(ii) Shear, e.g. (0,1) transforms to (3,1)	B1B1	2	One example or sensible explanation
	(iii) $\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$	M1 A1	2	Attempt to find DC (not CD) Obtain given answer
	(iv)	B1		Explicit check for $n = 1$ or $n = 2$
	$\mathbf{M}^{k} = \begin{pmatrix} 2^{k} 3(2^{k} - 1) \\ 0 & 1 \end{pmatrix} .$	M1		Induction hypothesis that result is true for \mathbf{M}^{k}
		M1		Attempt to multiply MM ^k or vice versa
	$\left(egin{array}{c} {k+1 \ 2 \ 0 \ 1 \ } \end{array} ight) \ .$	A1 A1		Element $3(2^{k+1}-1)$ derived correctly All other elements correct
		A1	6	Explicit statement of induction conclusion
			12	