

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

4753/1

Methods for Advanced Mathematics (C3)

Wednesday

25 MAY 2005

Afternoon

1 hour 30 minutes

Additional materials:

Answer booklet

Graph paper

MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

Section A (36 marks)

1 Solve the equation $|3x + 2| = 1$. [3]

2 Given that $\arcsin x = \frac{1}{6}\pi$, find x . Find $\arccos x$ in terms of π . [3]

3 The functions $f(x)$ and $g(x)$ are defined for the domain $x > 0$ as follows:

$$f(x) = \ln x, \quad g(x) = x^3.$$

Express the composite function $fg(x)$ in terms of $\ln x$.

State the transformation which maps the curve $y = f(x)$ onto the curve $y = fg(x)$. [3]

4 The temperature $T^\circ\text{C}$ of a liquid at time t minutes is given by the equation

$$T = 30 + 20e^{-0.05t}, \quad \text{for } t \geq 0.$$

Write down the initial temperature of the liquid, and find the initial rate of change of temperature.

Find the time at which the temperature is 40°C . [6]

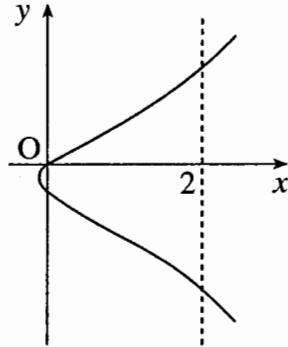
5 Using the substitution $u = 2x + 1$, show that $\int_0^1 \frac{x}{2x+1} dx = \frac{1}{4}(2 - \ln 3)$. [6]

6 A curve has equation $y = \frac{x}{2 + 3 \ln x}$. Find $\frac{dy}{dx}$. Hence find the exact coordinates of the stationary point of the curve. [7]

- 7 Fig. 7 shows the curve defined implicitly by the equation

$$y^2 + y = x^3 + 2x,$$

together with the line $x = 2$.



Not to scale

Fig. 7

Find the coordinates of the points of intersection of the line and the curve.

Find $\frac{dy}{dx}$ in terms of x and y . Hence find the gradient of the curve at each of these two points.

[8]

Section B (36 marks)

- 8 Fig. 8 shows part of the curve $y = x \sin 3x$. It crosses the x -axis at P. The point on the curve with x -coordinate $\frac{1}{6}\pi$ is Q.

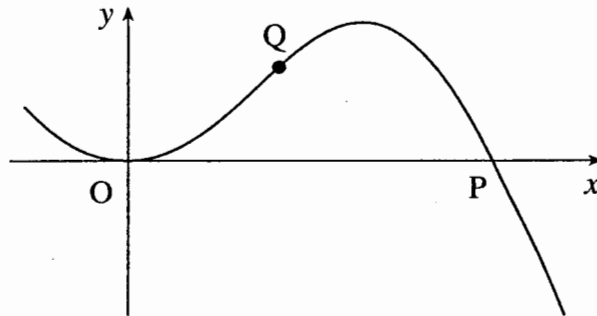


Fig. 8

- (i) Find the x -coordinate of P. [3]
- (ii) Show that Q lies on the line $y = x$. [1]
- (iii) Differentiate $x \sin 3x$. Hence prove that the line $y = x$ touches the curve at Q. [6]
- (iv) Show that the area of the region bounded by the curve and the line $y = x$ is $\frac{1}{72}(\pi^2 - 8)$. [7]

- 9 The function $f(x) = \ln(1 + x^2)$ has domain $-3 \leq x \leq 3$.

Fig. 9 shows the graph of $y = f(x)$.

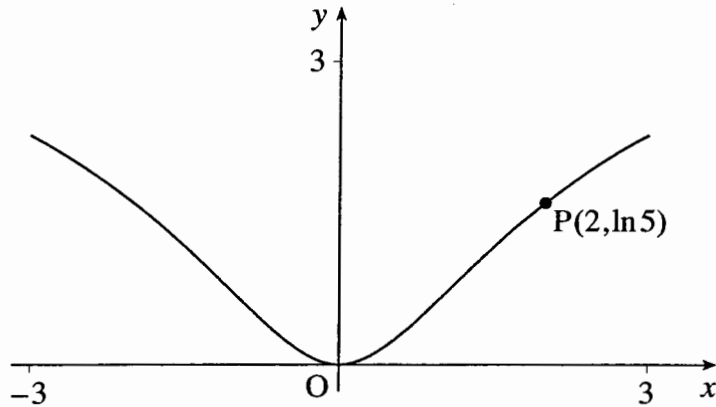


Fig. 9

- (i) Show algebraically that the function is even. State how this property relates to the shape of the curve. [3]
- (ii) Find the gradient of the curve at the point $P(2, \ln 5)$. [4]
- (iii) Explain why the function does not have an inverse for the domain $-3 \leq x \leq 3$. [1]

The domain of $f(x)$ is now restricted to $0 \leq x \leq 3$. The inverse of $f(x)$ is the function $g(x)$.

- (iv) Sketch the curves $y = f(x)$ and $y = g(x)$ on the same axes.

State the domain of the function $g(x)$.

Show that $g(x) = \sqrt{e^x - 1}$. [6]

- (v) Differentiate $g(x)$. Hence verify that $g'(\ln 5) = 1\frac{1}{4}$. Explain the connection between this result and your answer to part (ii). [5]