

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education**

MATHEMATICS

4733

Probability & Statistics 2

Wednesday

22 JUNE 2005

Afternoon

1 hour 30 minutes

Additional materials:

Answer booklet

Graph paper

List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

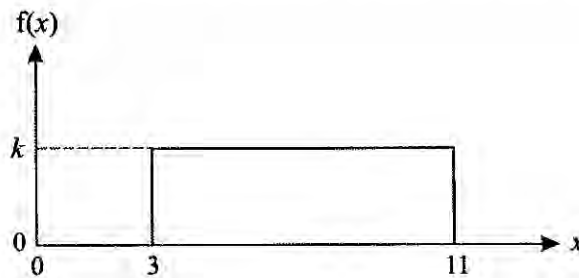
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

- 1 It is desired to obtain a random sample of 15 pupils from a large school. One pupil suggests listing all the pupils in the school in alphabetical order and choosing the first 15 names on the list.
- (i) Explain why this method is unsatisfactory. [2]
 - (ii) Suggest a better method. [2]
- 2 A continuous random variable has a normal distribution with mean 25.0 and standard deviation σ . The probability that any one observation of the random variable is greater than 20.0 is 0.75. Find the value of σ . [4]
- 3 (a) The random variable X has a $B(60, 0.02)$ distribution. Use an appropriate approximation to find $P(X \leq 2)$. [3]
- (b) The random variable Y has a $Po(30)$ distribution. Use an appropriate approximation to find $P(Y \leq 38)$. [5]
- 4 The height of sweet pea plants grown in a nursery is a random variable. A random sample of 50 plants is measured and is found to have a mean height 1.72 m and variance 0.0967 m^2 .
- (i) Calculate an unbiased estimate for the population variance of the heights of sweet pea plants. [2]
 - (ii) Hence test, at the 10% significance level, whether the mean height of sweet pea plants grown by the nursery is 1.8 m, stating your hypotheses clearly. [7]
- 5 The random variable W has the distribution $B(30, p)$.
- (i) Use the exact binomial distribution to calculate $P(W = 10)$ when $p = 0.4$. [2]
 - (ii) Find the range of values of p for which you would expect that a normal distribution could be used as an approximation to the distribution of W . [3]
 - (iii) Use a normal approximation to calculate $P(W = 10)$ when $p = 0.4$. [6]

- 6 A factory makes chocolates of different types. The proportion of milk chocolates made on any day is denoted by p . It is desired to test the null hypothesis $H_0 : p = 0.8$ against the alternative hypothesis $H_1 : p < 0.8$. The test consists of choosing a random sample of 25 chocolates. H_0 is rejected if the number of milk chocolates is k or fewer. The test is carried out at a significance level as close to 5% as possible.
- (i) Use tables to find the value of k , giving the values of any relevant probabilities. [3]
- (ii) The test is carried out 20 times, and each time the value of p is 0.8. Each of the tests is independent of all the others. State the expected number of times that the test will result in rejection of the null hypothesis. [2]
- (iii) The test is carried out once. If in fact the value of p is 0.6, find the probability of rejecting H_0 . [2]
- (iv) The test is carried out twice. Each time the value of p is equally likely to be 0.8 or 0.6. Find the probability that exactly one of the two tests results in rejection of the null hypothesis. [4]
- 7 The continuous random variable X has the probability density function shown in the diagram.



- (i) Find the value of the constant k . [2]
- (ii) Write down the mean of X , and use integration to find the variance of X . [5]
- (iii) Three observations of X are made. Find the probability that $X < 9$ for all three observations. [3]
- (iv) The mean of 32 observations of X is denoted by \bar{X} . State the approximate distribution of \bar{X} , giving its mean and variance. [3]

[Question 8 is printed overleaf.]

8 In excavating an archaeological site, Roman coins are found scattered throughout the site.

- (i) State two assumptions needed to model the number of coins found per square metre of the site by a Poisson distribution. [2]

Assume now that the number of coins found per square metre of the site can be modelled by a Poisson distribution with mean λ .

- (ii) Given that $\lambda = 0.75$, calculate the probability that exactly 3 coins are found in a region of the site of area 7.20 m^2 . [3]

A test is carried out, at the 5% significance level, of the null hypothesis $\lambda = 0.75$, against the alternative hypothesis $\lambda > 0.75$, in Region LVI which has area 4 m^2 .

- (iii) Determine the smallest number of coins that, if found in Region LVI, would lead to rejection of the null hypothesis, stating also the values of any relevant probabilities. [4]
- (iv) Given that, in fact, $\lambda = 1.2$ in Region LVI, find the probability that the test results in a Type II error. [3]