Mark Scheme 4754
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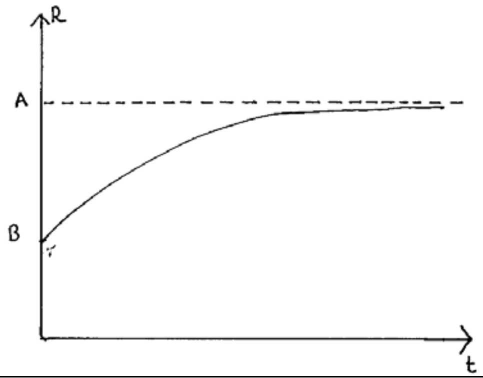
1 $\sin x - \sqrt{3}\cos x = R\sin(x-\alpha)$ $= R(\sin x\cos\alpha - \cos x\sin\alpha)$ $\Rightarrow R\cos\alpha = 1,\ R\sin\alpha = \sqrt{3}$ $\Rightarrow R^2 = 1^2 + (\sqrt{3})^2 = 4,\ R = 2$ $\tan\alpha = \sqrt{3}/1 = \sqrt{3} \Rightarrow \alpha = \pi/3$ $\Rightarrow \sin x - \sqrt{3}\cos x = 2\sin(x-\pi/3)$ x coordinate of P is when $x - \pi/3 = \pi/2$ $\Rightarrow x = 5\pi/6$ $y = 2$ So coordinates are $(5\pi/6, 2)$	B1 M1 A1 M1 A1ft B1ft [6]	$R = 2$ $\tan\alpha = \sqrt{3}$ or $\sin\alpha=\sqrt{3}$/their R or $\cos\alpha=1$/their R $\alpha = \pi/3$, 60° or 1.05 (or better) radians www Using x-their $\alpha=\pi/2$or 90° $\alpha\neq 0$ exact radians only (not $\pi/2$) their R (exact only)
2(i) $\dfrac{3+2x^2}{(1+x)^2(1-4x)} = \dfrac{A}{1+x} + \dfrac{B}{(1+x)^2} + \dfrac{C}{1-4x}$ $\Rightarrow 3+2x^2 = A(1+x)(1-4x) + B(1-4x) + C(1+x)^2$ $x = -1 \Rightarrow 5 = 5B \Rightarrow B = 1$ $x = ¼ \Rightarrow 3\frac{1}{8} = \frac{25}{16}C \Rightarrow C = 2$ coeff[t] of x^2: $2 = -4A + C \Rightarrow A = 0$.	M1 B1 B1 E1 [4]	Clearing fractions (or any 2 correct equations) $B = 1$ www $C = 2$ www $A = 0$ needs justification
(ii) $(1+x)^{-2} = 1 + (-2)x + (-2)(-3)x^2/2! + \ldots$ $= 1 - 2x + 3x^2 + \ldots$ $(1-4x)^{-1} = 1 + (-1)(-4x) + (-1)(-2)(-4x)^2/2! + \ldots$ $= 1 + 4x + 16x^2 + \ldots$ $\dfrac{3+2x^2}{(1+x)^2(1-4x)} = (1+x)^{-2} + 2(1-4x)^{-1}$ $\approx 1 - 2x + 3x^2 + 2(1 + 4x + 16x^2)$ $= 3 + 6x + 35x^2$	M1 A1 A1 A1ft [4]	Binomial series (coefficients unsimplified - for either) or $(3+2x^2)(1+x)^{-2}(1-4x)^{-1}$ expanded theirA,B,C and their expansions
3 $\sin(\theta+\alpha) = 2\sin\theta$ $\Rightarrow \sin\theta\cos\alpha + \cos\theta\sin\alpha = 2\sin\theta$ $\Rightarrow \tan\theta\cos\alpha + \sin\alpha = 2\tan\theta$ $\Rightarrow \sin\alpha = 2\tan\theta - \tan\theta\cos\alpha$ $= \tan\theta\,(2-\cos\alpha)$ $\Rightarrow \tan\theta = \dfrac{\sin\alpha}{2-\cos\alpha}$ * $\sin(\theta + 40°) = 2\sin\theta$ $\Rightarrow \tan\theta = \dfrac{\sin 40}{2-\cos 40} = 0.5209$ $\Rightarrow \theta = 27.5°, 207.5°$	M1 M1 M1 E1 M1 A1 A1 [7]	Using correct Compound angle formula in a valid equation dividing by $\cos\theta$ collecting terms in $\tan\theta$ or $\sin\theta$ or dividing by $\tan\theta$ oe www (can be all achieved for the method in reverse) $\tan\theta = \dfrac{\sin 40}{2-\cos 40}$ -1 if given in radians -1 extra solutions in the range

4 (a) $\frac{dx}{dt} = k\sqrt{x}$	M1 A1 [2]	$\frac{dx}{dt} = \ldots$ $k\sqrt{x}$
(b) $\frac{dy}{dt} = \frac{10000}{\sqrt{y}}$ $\Rightarrow \int \sqrt{y}\,dy = \int 10000\,dt$ $\Rightarrow \frac{2}{3}y^{\frac{3}{2}} = 10000t + c$ When $t = 0$, $y = 900 \Rightarrow 18000 = c$ $\Rightarrow y = [\frac{3}{2}(10000t + 18000)]^{\frac{2}{3}}$ $= (1500(10t+18))^{\frac{2}{3}}$ When $t = 10$, $y = 3152$	M1 A1 B1 A1 M1 A1 [6]	separating variables condone omission of c evaluating constant for their integral any correct expression for y = for method allow substituting t=10 in their expression cao
5 (i) $\int xe^{-2x}dx$ let $u = x$, $dv/dx = e^{-2x}$ $\Rightarrow v = -½\, e^{-2x}$ $= -\frac{1}{2}xe^{-2x} + \int \frac{1}{2}e^{-2x}dx$ $= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + c$ $= -\frac{1}{4}e^{-2x}(1+2x) + c$ * or $\frac{d}{dx}[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + c] = -\frac{1}{2}e^{-2x} + xe^{-2x} + \frac{1}{2}e^{-2x}$ $= xe^{-2x}$	M1 A1 E1 M1 A1 E1 [3]	Integration by parts with $u = x$, $dv/dx = e^{-2x}$ $= -\frac{1}{2}xe^{-2x} + \int \frac{1}{2}e^{-2x}dx$ condone omission of c product rule
(ii) $V = \int_0^2 \pi y^2 dx$ $= \int_0^2 \pi (x^{1/2}e^{-x})^2 dx$ $= \pi \int_0^2 xe^{-2x}dx$ $= \pi\left[-\frac{1}{4}e^{-2x}(1+2x)\right]_0^2$ $= \pi(-¼\, e^{-4}.5 + ¼)$ $= \frac{1}{4}\pi(1 - \frac{5}{e^4})$ *	M1 A1 DM1 E1 [4]	Using formula condone omission of limits $y^2 = xe^{-2x}$ condone omission of limits and π condone omission of π (need limits)

Section B

6 (i) At E, $\theta = 2\pi$ $\Rightarrow$ $x = a(2\pi - \sin 2\pi) = 2a\pi$ So OE = $2a\pi$. Max height is when $\theta = \pi$ $\Rightarrow$ $y = a(1 - \cos \pi) = 2a$	M1 A1 M1 A1 [4]	$\theta=\pi$, 180°, $\cos\theta=-1$
(ii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{a\sin\theta}{a(1-\cos\theta)}$ $= \frac{\sin\theta}{(1-\cos\theta)}$	M1 M1 A1 [3]	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ for theirs $\frac{d}{d\theta}(\sin\theta) = \cos\theta,\ \frac{d}{d\theta}(\cos\theta) = -\sin\theta$ both or equivalent www condone uncancelled a
(iii) $\tan 30° = 1/\sqrt{3}$ $\Rightarrow$ $\frac{\sin\theta}{(1-\cos\theta)} = \frac{1}{\sqrt{3}}$ $\Rightarrow$ $\sin\theta = \frac{1}{\sqrt{3}}(1-\cos\theta)$ * When $\theta = 2\pi/3$, $\sin\theta = \sqrt{3}/2$ $(1 - \cos\theta)/\sqrt{3} = (1 + ½)/\sqrt{3} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$ BF = $a(1 + ½) = 3a/2$* OF = $a(2\pi/3 - \sqrt{3}/2)$	M1 E1 M1 E1 E1 B1 [6]	Or gradient=$1/\sqrt{3}$ $\sin\theta = \sqrt{3}/2$, $\cos\theta = -½$ or equiv.
(iv) BC = $2a\pi - 2a(2\pi/3 - \sqrt{3}/2)$ $= a(2\pi/3 + \sqrt{3})$ AF = $\sqrt{3} \times 3a/2 = 3\sqrt{3}a/2$ AD = BC + 2AF $= a(2\pi/3 + \sqrt{3} + 3\sqrt{3})$ $= a(2\pi/3 + 4\sqrt{3})$ $= 20$ $\Rightarrow a = 2.22$ m	B1ft M1 A1 M1 A1 [5]	their OE -2their OF

7 (i) AE = $\sqrt{15^2 + 20^2 + 0^2} = 25$	M1 A1 [2]	
(ii) $\overrightarrow{AE} = \begin{pmatrix} 15 \\ -20 \\ 0 \end{pmatrix} = 5\begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$ Equation of BD is $\mathbf{r} = \begin{pmatrix} -1 \\ -7 \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$ BD = 15 $\Rightarrow \lambda = 3$ $\Rightarrow$ D is (8, –19, 11)	M1 A1 M1 A1cao [4]	Any correct form or $\mathbf{r} = \begin{pmatrix} -1 \\ -7 \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} 15 \\ -20 \\ 0 \end{pmatrix}$ λ = 3 or 3/5 as appropriate
(iii) At A: –3 × 0 + 4 × 0 + 5 × 6 = 30 At B: –3 × (–1) + 4 × (–7) + 5 × 11 = 30 At C: –3 × (–8) + 4 × (–6) + 5 × 6 = 30 Normal is $\begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$	M1 A2,1,0 B1 [4]	One verification (OR B1 Normal, M1 scalar product with 1 vector in the plane, A1two correct, A1 verification with a point OR M1 vector form of equation of plane eg r=0i+0j+6k+μ(i+7j-5k)+ν(8i+6j+0k) M1 elimination of both parameters A1 equation of plane B1 Normal *)
(iv) $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}.\overrightarrow{AE} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}.\begin{pmatrix} 15 \\ -20 \\ 0 \end{pmatrix} = 60 - 60 = 0$ $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}.\overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}.\begin{pmatrix} -1 \\ -7 \\ 5 \end{pmatrix} = -4 - 21 + 25 = 0$ $\Rightarrow \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$ is normal to plane Equation is $4x + 3y + 5z = 30$.	M1 E1 M1 A1 [4]	scalar product with one vector in plane = 0 scalar product with another vector in plane = 0 $4x + 3y + 5z = \ldots$ 30 OR as * above OR M1 for subst 1 point in 4x+3y+5z= ,A1 for subst 2 further points =30 A1 correct equation , B1 Normal
(v) Angle between planes is angle between normals $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$ $\cos\theta = \dfrac{4\times(-3)+3\times4+5\times5}{\sqrt{50}\times\sqrt{50}} = \dfrac{1}{2}$ $\Rightarrow \theta = 60°$	M1 M1 A1 A1 [4]	 Correct method for any 2 vectors their normals only (rearranged) or 120° cao

	Comprehension Paper 2		
Qu	**Answer**	**Mark**	**Comment**
1.	$\left(26+\frac{385}{1760}\right)\times 4$ minutes 1 hour 44 minutes 52.5 seconds	M1 A1	Accept all equivalent forms, with units. Allow ….52 and 53 seconds.
2.	$R = 259.6 - 0.391(T-1900)$ $\therefore\ 259.6 - 0.391(T-1900) = 0$ $\Rightarrow T = 2563.9$ *R* will become negative in 2563	M1 A1 A1	R=0 and attempting to solve. T=2563,2564,2563.9…any correct cao
3.	The value of *L* is 120.5 and this is over 2 hours or (120 minutes)	E1	or R>120.5minutes or showing there is no solution for $120=120.5+54.5e^{-...}$
4.(i)	Substituting $t=0$ in $R = L+(U-L)e^{-kt}$ gives $R = L+(U-L)\times 1$ $= U$	M1 A1 E1	$e^0 = 1$
4.(ii)	As $t\to\infty$, $e^{-kt}\to 0$ and so $R\to L$	M1 E1	
5.(i)		M1 A1 A1	Increasing curve Asymptote *A* and *B* marked correctly
5.(ii)	Any field event: long jump, high jump, triple jump, pole vault, javelin, shot, discus, hammer, etc.	B1	
6.(i)	$t = 104$	B1	
6.(ii)	$R = 115+(175-115)e^{-0.0467t^{0.797}}$ $R = 115+60\times e^{-0.0467\times 104^{0.797}}$ $R = 115+60\times e^{-1.892}$ $R = 124.047...$ 2 hours 4 minutes 3 seconds	M1 A1	Substituting their t 124, 124.05, etc.