Mark Scheme 4727 June 2006

	1	
1 (a) Identity = 1+0i	B1	For correct identity. Allow 1
Inverse = $\frac{1}{1+2i}$	B1	For $\frac{1}{1+2i}$ seen or implied
$= \frac{1}{1+2i} \times \frac{1-2i}{1-2i} = \frac{1}{5} - \frac{2}{5}i$	B1 3	For correct inverse AEFcartesian
(b) Identity = $ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} $	B1	For correct identity
Inverse = $\begin{pmatrix} -3 & 0 \\ 0 & 0 \end{pmatrix}$	B1 2	For correct inverse
	5	
2 (a) $(z_1 z_2 =) 6e^{\frac{5}{12}\pi i}$	B1	For modulus = 6
$(z_1 z_2 =) 6e^{-z}$	B1	For argument = $\frac{5}{12}\pi$
$\left(\frac{z_1}{z_2} = \frac{2}{3} e^{-\frac{1}{12}\pi i} = \right) \frac{2}{3} e^{\frac{23}{12}\pi i}$	M1	For subtracting arguments
$\left(z_2 - 3 - 3 - 1\right)$	A1 4	For correct answer
(b) $\left(w^{-5} = \right) 2^{-5} \operatorname{cis} \left(-\frac{5}{8}\pi\right)$	M1	For use of de Moivre
	A1	For $-\frac{5}{8}\pi$ seen or implied
$=\frac{1}{32}\left(\cos\frac{11}{8}\pi+i\sin\frac{11}{8}\pi\right)$	A1 3	For correct answer (allow 2^{-5} and $cis \frac{11}{8}\pi$)
	7	

3 EITHER $\mathbf{c} - \mathbf{a} = \pm [11, 3, -2]$ ($\mathbf{c} - \mathbf{a}) \times [8, 3, -6]$ M1* $\mathbf{n} = \pm [-12, 50, 9]$ A1 $\sqrt{}$ For exterp tat vector product of $\mathbf{c} - \mathbf{a}$ and $[8, 3, -6]$ For obtaining \mathbf{n} . f.t. from incorrect $\mathbf{c} - \mathbf{a}$ For dividing $ \mathbf{n} $ by magnitude of $[8, 3, -6]$ For correct distance CAO For eal $\pm [11, 3, -2]$ For either magnitude correct $\mathbf{c} - \mathbf{a} - \mathbf{a} = \pm [11, 3, -2]$ For correct distance CAO For vector joining lines For attempt at vector product of $\mathbf{c} - \mathbf{a}$ and $[8, 3, -6]$ For dividing $ \mathbf{n} $ by magnitude of $[8, 3, -6]$ For correct distance CAO For either magnitude correct For correct distance CAO For vector joining lines For obtaining \mathbf{n} . f.t. from incorrect $\mathbf{c} - \mathbf{a}$ For dividing $ \mathbf{n} $ by magnitude of $[8, 3, -6]$ For correct distance CAO For vector joining lines For correct distance CAO For either magnitude correct For correct distance CAO For vector joining lines For obtaining \mathbf{n} . f.t. from incorrect $\mathbf{c} - \mathbf{a}$ and $[8, 3, -6]$ For other magnitude correct For correct distance CAO For vector joining lines For correct distance CAO For vector joining lines For dividing $ \mathbf{n} $ by magnitude of $[8, 3, -6]$ For other magnitude correct For correct distance CAO For vector joining lines For other magnitude correct For attempt at scalar product of $\mathbf{c} - \mathbf{a}$ and $[8, 3, -6]$ For using trigonometry for perpendicular distance For using trigonometry for perpendicular distance For attempt at scalar product of $\mathbf{c} - \mathbf{a}$ and $[8, 3, -6]$ For finding projection of $\mathbf{c} - \mathbf{a}$ and $[8, 3, -6]$ For finding projection of $\mathbf{c} - \mathbf{a}$ and $[8, 3, -6]$ For finding projection of $\mathbf{c} - \mathbf{a}$ and $[8, 3, -6]$ fit. from incorrect $\mathbf{c} - \mathbf{a}$ For using Pythagoras for perpendicular distance For using Pythagoras for perpendicular distance For latempt at scalar product of $\mathbf{c} - \mathbf{a}$ and $[8, 3, -6]$ For finding a vector from $\mathbf{C} - \mathbf{a}$ For orient distance CAO For latempt at vector joining lines For attempt at vector joining lines Fo		1	<u>†</u>
$\begin{array}{lll} \mathbf{n} = \pm[-12,50,9] & \mathbf{A}1 \ \ \\ d = \frac{ \mathbf{n} }{\ 8,3,-6\ } & \mathbf{M}1 \\ (dep^*) & \mathbf{For obtaining n. f.t. from incorrect } \mathbf{c} - \mathbf{a} \\ & = \frac{\sqrt{2725}}{\sqrt{109}} & \mathbf{A}1 \\ (dep^*) & \mathbf{For obtaining n. f.t. from incorrect } \mathbf{c} - \mathbf{a} \\ & = \frac{\sqrt{109}}{\sqrt{109}} & \mathbf{A}1 \\ (c - \mathbf{a}) \cdot [8,3,-6] & \mathbf{M}1^* & \mathbf{For correct distance CAO} \\ & \mathbf{COR} \ \ \mathbf{c} - \mathbf{a} = \pm[11,3,-2] & \mathbf{B}1 \\ & \mathbf{cos} \ \ 0 = \pm \frac{109}{\sqrt{134}\sqrt{109}} = \pm \sqrt{\frac{109}{134}} & \mathbf{A}1 \ \ \\ & \mathbf{M}1^* & \mathbf{For correct cos} \ \ \mathbf{AEF. f.t. from incorrect cos} \\ & \mathbf{A}1 \ \ A$	3 EITHER $c-a = \pm [11, 3, -2]$	B1	For vector joining lines
$ \begin{array}{lll} \mathbf{n} = \pm [-12, 50, 9] \\ d = \frac{ \mathbf{n} }{\ [8, 3, -6] } \\ & = \frac{\sqrt{2725}}{\sqrt{109}} \\ (d =) 5 \\ & = \frac{109}{\sqrt{134}\sqrt{109}} = \pm \sqrt{\frac{109}{134}} \\ & = \frac{109}{\sqrt{134}\sqrt{109}} = \pm \sqrt{\frac{109}{134}} \\ & = \frac{109}{\sqrt{109}} = \pm \sqrt{\frac{109}{134}} \\ & = \frac{109}{\sqrt{109}} = \pm \sqrt{\frac{109}{134}} \\ & = \frac{109}{\sqrt{109}} = \sqrt{\frac{109}{109}} \\ & = \frac{109}{\sqrt{109}} = \frac{1$	$(\mathbf{c} - \mathbf{a}) \times [8, 3, -6]$	M1*	· · · · · · · · · · · · · · · · · · ·
$=\frac{\sqrt{2725}}{\sqrt{109}}$ $(d =) 5$ $OR \ c-a = \pm \{11, 3, -2\}$ $(c-a) \cdot [8, 3, -6]$ $DR \ c = \pm \frac{109}{\sqrt{134}\sqrt{109}} = \pm \sqrt{\frac{109}{134}}$ $d = \sqrt{134}\sin 0$ $d = \sqrt{1}$ $d $	$\mathbf{n} = \pm [-12, 50, 9]$	A1 √	For obtaining n . f.t. from incorrect $c-a$
$ \begin{array}{c} (d=) \ 5 \\ \hline OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ cos \ \theta=\pm \frac{109}{\sqrt{134}\sqrt{109}} = \pm \sqrt{\frac{109}{134}} \\ \hline \\ d=\sqrt{134}\sin 0 \\ \hline \\ (d=) \ 5 \\ \hline OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ (d=) \ 5 \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ (d=) \ 5 \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ (d=) \ 5 \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ (d=) \ 5 \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (d-a),[8,3,-6] \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (d-a),[8,3,-6] \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (d-a),[8,3,-6] \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ \hline \\ OR \ c-a=\pm 1 ,3,-6 \\ \hline \\ $	$d = \frac{ \mathbf{n} }{ [8, 3, -6] }$		For dividing $ \mathbf{n} $ by magnitude of $[8, 3, -6]$
$ \begin{array}{c} (d=) \ 5 \\ \hline OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ cos \ \theta=\pm \frac{109}{\sqrt{134}\sqrt{109}} = \pm \sqrt{\frac{109}{134}} \\ \hline \\ d=\sqrt{134}\sin 0 \\ \hline \\ (d=) \ 5 \\ \hline OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ (d=) \ 5 \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ (d=) \ 5 \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ (d=) \ 5 \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (c-a),[8,3,-6] \\ \hline \\ (d=) \ 5 \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (d-a),[8,3,-6] \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (d-a),[8,3,-6] \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ (d-a),[8,3,-6] \\ \hline \\ OR \ c-a=\pm 1 ,3,-2 \\ \hline \\ OR \ c-a=\pm 1 ,3,-6 \\ \hline \\ $	$=\frac{\sqrt{2725}}{\sqrt{109}}$	A1	For either magnitude correct
$(\mathbf{c}-\mathbf{a}).[8,3,-6]$ $\cos\theta = \pm \frac{109}{\sqrt{134}\sqrt{109}} = \pm \sqrt{\frac{109}{134}}$ $d = \sqrt{134}\sin\theta$ $(d =) 5$ $OR \mathbf{c}-\mathbf{a} = \pm [11,3,-2]$ $(\mathbf{c}-\mathbf{a}).[8,3,-6]$ $M1^*$ $(d =) 5$ $OR \mathbf{c}-\mathbf{a} = \pm [11,3,-2]$ $(\mathbf{c}-\mathbf{a}).[8,3,-6]$ $M1^*$ \mathbf{M}^* \mathbf{M}	•	A1	For correct distance CAO
$\cos\theta = \pm \frac{109}{\sqrt{134}\sqrt{109}} = \pm \sqrt{\frac{109}{134}}$ $d = \sqrt{134}\sin\theta$ $(d =) 5$ $OR \mathbf{c} - \mathbf{a} = \pm [11, 3, -2]$ $(\mathbf{c} - \mathbf{a}) . [8, 3, -6]$ $x = \frac{109}{\sqrt{109}} = \sqrt{109}$ $d = \sqrt{134-109}$ $(d =) 5$ $OR \mathbf{c} - \mathbf{a} = \pm [11, 3, -2]$ $(\mathbf{c} - \mathbf{a}) . [8, 3, -6]$ \mathbf{M}^{1} (dep^{*}) $A1 \downarrow \mathbf{M}^{2}$ (dep^{*}) $A1 \downarrow \mathbf{M}^{2}$ (dep^{*}) $A1 \downarrow \mathbf{M}^{2}$ (dep^{*}) $A1 \downarrow \mathbf{M}^{2}$ (dep^{*}) $A1 \downarrow \mathbf{M}^{3}$ (dep^{*}) $A1 \downarrow \mathbf{M}^{2}$ (dep^{*}) $A1 \downarrow \mathbf{M}^{3}$ (dep^{*}) $A1 \downarrow \mathbf{M}^{4}$ $(dep^{*}$	$OR \ \mathbf{c-a} = \pm [11, 3, -2]$	B1	For vector joining lines
For using trigonometry for perpendicular distance For correct expression for d in terms of θ For correct distance CAO For each d and d for each d for interms of d for correct distance CAO For vector joining lines For attempt at scalar product of d for or and d for finding a vector from d for finding and d for d	$(\mathbf{c} - \mathbf{a}) \cdot [8, 3, -6]$	M1*	· · · · · · · · · · · · · · · · · · ·
	$\cos \theta = \pm \frac{109}{\sqrt{134}\sqrt{109}} = \pm \sqrt{\frac{109}{134}}$	A1 √	
	$d = \sqrt{134} \sin \theta$	(dep*)	distance
$ \begin{array}{c} \textit{OR} \mathbf{c-a} = \pm [11, 3, -2] \\ (\mathbf{c-a}) \cdot [8, 3, -6] \\ \\ \textit{x} = \frac{109}{\sqrt{109}} = \sqrt{109} \\ \\ \textit{d} = \sqrt{134 - 109} \\ \\ \textit{(d =) 5} \\ \\ \textit{CP} \cdot [8, 3, -6] = 0 \\ \\ \textit{t} = \pm 1 \textit{OR} \textit{P} = (9, 5, -1) \\ \textit{d} = \sqrt{3^2 + 0^2 + 4^2} \\ \textit{(d =) 5} \\ \\ \textit{(d =) 5} \\ \\ \textit{A1} \\ \textit{M1*} \\ \textit{(d =) 5} \\ \\ \textit{Por inding projection of } \mathbf{c-a} \text{ and } [8, 3, -6] \\ \textit{f.t. from incorrect } \mathbf{c-a} \\ \textit{For using Pythagoras for perpendicular distance} \\ \textit{For correct expression for } \textit{d} \\ \textit{For correct distance } \mathbf{CAO} \\ \textit{For finding a vector from } \textit{C}(12, 5, 3) \\ \textit{to a point on the line} \\ \textit{For correct point. f.t. from incorrect } \mathbf{CP} \\ \textit{For correct expression for } \textit{d} \\ \textit{(d =) 5} \\ \textit{M1} \\ \textit{(dep*)} \\ \textit{A1} \\ \textit{(dep*)} \\ \textit{(d =) 5} \\$	(d =) 5	Δ1	•
$(\mathbf{c}-\mathbf{a}) \cdot [8,3,-6]$ $x = \frac{109}{\sqrt{109}} = \sqrt{109}$ $A1 \sqrt{\begin{array}{c} \text{For attempt at scalar product of } \mathbf{c} - \mathbf{a} \text{ and } [8,3,-6] \\ \text{For finding projection of } \mathbf{c} - \mathbf{a} \text{ onto } [8,3,-6] \\ \text{f.t. from incorrect } \mathbf{c} - \mathbf{a} \\ \text{M1} \\ \text{(dep*)} \\ \text{A1} \\ \text{(dep*)} \\ \text{A2} \\ \text{(dep*)} \\ \text{(dep*)} \\ \text{A3} \\ \text{(dep*)} \\ \text{A4} \\ \text{(dep*)} \\ \text{A5} \\ \text{(dep*)} \\ \text{A6} \\ \text{(dep*)} \\ \text{A7} \\ \text{(dep*)} \\ \text{A8} \\ \text{(dep*)} \\ \text{A9} \\ \text{(dep*)} \\ \text{A9} \\ \text{(dep*)} \\ \text{A1} \\ \text{(dep*)} \\ \text{A2} \\ \text{(dep*)} \\ \text{A3} \\ \text{(dep*)} \\ \text{A4} \\ \text{(dep*)} \\ \text{A5} \\ \text{(dep*)} \\ \text{A6} \\ \text{(dep*)} \\ \text{A7} \\ \text{(dep*)} \\ \text{A8} \\ \text{(dep*)} \\ \text{A9} \\ \text{(dep*)} \\ \text{A1} \\ \text{(dep*)} \\ \text{A2} \\ \text{(dep*)} \\ \text{A3} \\ \text{(dep*)} \\ \text{A4} \\ \text{(dep*)} \\ \text{A5} \\ \text{(dep*)} \\ \text{A6} \\ \text{(dep*)} \\ \text{A7} \\ \text{(dep*)} \\ \text{A8} \\ \text{(dep*)} \\ \text{A9} \\ \text{(dep*)} \\ \text{A9} \\ \text{(dep*)} \\ \text{A1} \\ \text{(dep*)} \\ \text{A2} \\ \text{(dep*)} \\ \text{A3} \\ \text{(dep*)} \\ \text{A4} \\ \text{(dep*)} \\ \text{A5} \\ \text{(dep*)} \\ \text{A6} \\ \text{(dep*)} \\ \text{A7} \\ \text{(dep*)} \\ \text{A8} \\ \text{(dep*)} \\ \text{A9} \\ \text{(dep*)} \\ \text{A9} \\ \text{(dep*)} \\ \text{A1} \\ \text{(dep*)} \\ \text{A2} \\ \text{(dep*)} \\ \text{A3} \\ \text{(dep*)} \\ \text{A4} \\ \text{(dep*)} \\ \text{A5} \\ \text{(dep*)} \\ \text{A6} \\ \text{(dep*)} \\ \text{A7} \\ \text{(dep*)} \\ \text{A8} \\ \text{(dep*)} \\ \text{A9} \\ \text{(dep*)} \\ \text{(dep*)} \\ \text{A9} \\ \text{(dep*)} \\ \text{(dep*)}$			
$d = \sqrt{134 - 109}$ $d = \sqrt{134 - 109}$ $(d =) 5$ $OR \mathbf{CP} = \pm [-11 + 8t, -3 + 3t, 2 - 6t]$ $\mathbf{CP} \cdot [8, 3, -6] = 0$ $t = \pm 1 OR P = (9, 5, -1)$ $d = \sqrt{3^2 + 0^2 + 4^2}$ $(d =) 5$ $\mathbf{M1}$ (dep^*) $d = \sqrt{3}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M5}$ $\mathbf{M4}$ $\mathbf{M5}$ $\mathbf{M6}$ $\mathbf{M6}$ $\mathbf{M6}$ $\mathbf{M6}$ $\mathbf{M7}$ $\mathbf{M1}$ $\mathbf{M9}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M1}$ $\mathbf{M5}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M1}$ $\mathbf{M5}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M1}$ $\mathbf{M5}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M5}$ $\mathbf{M5}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M5}$ $\mathbf{M5}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M5}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M5}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M5}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M5}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M5}$ $\mathbf{M5}$ $\mathbf{M5}$ $\mathbf{M5}$ $\mathbf{M6}$ $\mathbf{M6}$ $\mathbf{M6}$ $\mathbf{M7}$ $\mathbf{M9}$ $\mathbf{M9}$ $\mathbf{M9}$ $\mathbf{M9}$ $\mathbf{M9}$ $\mathbf{M9}$ $\mathbf{M9}$ $M9$			For attempt at scalar product of $\mathbf{c}-\mathbf{a}$ and
$d = \sqrt{134 - 109}$ $(d =) 5$ $OR \mathbf{CP} = \pm [-11 + 8t, -3 + 3t, 2 - 6t]$ $t = \pm 1 OR P = (9, 5, -1)$ $d = \sqrt{3^2 + 0^2 + 4^2}$ $(d =) 5$ $\mathbf{M1}$ $(d =) 5$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M4}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M5}$ $\mathbf{M1}$ $\mathbf{M6}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M4}$ $\mathbf{M5}$ $\mathbf{M1}$ $\mathbf{M6}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M4}$ $\mathbf{M5}$ $\mathbf{M1}$ $\mathbf{M6}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M4}$ $\mathbf{M4}$ $\mathbf{M5}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M1}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M3}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M1}$ $\mathbf{M2}$ $\mathbf{M3}$ $$	$x = \frac{109}{\sqrt{109}} = \sqrt{109}$	A1 √	[8, 3, -6]
$ \begin{array}{c} (\textit{d} =) \ 5 \\ OR \ \ CP = \pm [-11+8t, -3+3t, 2-6t] \\ CP \cdot [8, 3, -6] = 0 \\ t = \pm 1 \ \ OR \ \ P = (9, 5, -1) \\ d = \sqrt{3^2+0^2+4^2} \\ (\textit{d} =) \ 5 \\ \end{array} \begin{array}{c} \text{A1} \\ \text{B1} \\ \text{M1*} \\ \text{M2} \\ \text{For correct distance CAO} \\ \text{For inding a vector from } \ \ C(12, 5, 3) \\ \text{to a point on the line} \\ \text{For using scalar product for perpendicularity} \\ \text{For correct point. f.t. from incorrect CP} \\ \text{M1} \\ \text{(dep*)} \\ \text{A1} \\ \text{A1} \\ \text{6} \\ \end{array} \begin{array}{c} \text{For finding magnitude of CP} \\ \text{For correct expression for } \ \ d \\ \text{For correct distance CAO} \\ \text{SR Obtain} \\ \text{CP} = [11, 3, -2] - [8, 3, -6] = \pm [3, 0, 4] \ \ B1 \\ \text{Verify } [3, 0, 4] \cdot [8, 3, -6] = 0 \\ \text{M1*} \\ d = \sqrt{3^2+0^2+4^2} = 5 \\ \text{M1(dep*) A1 A1} \\ \text{(maximum 5 / 6)} \end{array} $	$d = \sqrt{134 - 109}$	(dep*)	For using Pythagoras for perpendicular distance
to a point on the line CP \cdot [8, 3, -6] = 0 M1* The second of the line shows the line shows to a point on the line shows to a point on the line for using scalar product for perpendicularity A1 $\sqrt{\frac{1}{2} + 0^2 + 4^2}$ For correct point. f.t. from incorrect CP M1 $\sqrt{\frac{1}{2} + 0^2 + 4^2}$ For correct expression for d For correct distance CAO SR Obtain CP = [11, 3, -2] - [8, 3, -6] = \pm [3, 0, 4] \text{ B1} Verify [3, 0, 4] \cdot [8, 3, -6] = 0 M1* $d = \sqrt{3^2 + 0^2 + 4^2} = 5$ M1(dep*) A1 A1 (maximum 5 / 6)	(d =) 5	A1	•
CP. $[8,3,-6]=0$ $t=\pm 1 \ OR \ P=(9,5,-1)$ $d=\sqrt{3^2+0^2+4^2}$ A1 \((dep^*) \) A1 \((dep^*) \) A1 \((dep^*) \) A1 \((dep^*	•	B1	, , ,
$t = \pm 1 \ OR \ P = (9, 5, -1)$ $d = \sqrt{3^2 + 0^2 + 4^2}$ $(d =) \ 5$ A1 \(\text{M1} \) $(dep^*) \(A1 \) A1 \(6 \) For correct point. f.t. from incorrect CP For finding magnitude of CP For correct expression for d For correct distance CAO SR Obtain CP = [11, 3, -2] - [8, 3, -6] = \pm [3, 0, 4] \ B1 Verify \ [3, 0, 4] \cdot [8, 3, -6] = 0 \ M1^* d = \sqrt{3^2 + 0^2 + 4^2} = 5 \ M1(dep^*) \ A1 \ A1 \ (maximum 5 / 6)$	$\mathbf{CP} \cdot [8, 3, -6] = 0$	M1*	For using scalar product for
$d = \sqrt{3^2 + 0^2 + 4^2}$ $(d =) 5$ $M1 \\ (dep*) \\ A1 \\ A1 $ $A1 $ $A2 $ $A2 $ $A2 $ $A2 $ $A2 $ $A3 $ $A2 $ $A3 $ $A4 $ $A1 $ $A2 $ $A3 $ $A4 $	$t = \pm 1$ OR $P = (9, 5, -1)$	A1 √	1
(d =) 5 A1 6 For correct distance CAO SR Obtain $CP = [11, 3, -2] - [8, 3, -6] = \pm [3, 0, 4]$ B1 Verify $[3, 0, 4] \cdot [8, 3, -6] = 0$ M1* $d = \sqrt{3^2 + 0^2 + 4^2} = 5$ M1(dep*) A1 A1 (maximum 5 / 6)	$d = \sqrt{3^2 + 0^2 + 4^2}$		•
SR Obtain CP = [11, 3, -2] - [8, 3, -6] = \pm [3, 0, 4] B1 Verify [3, 0, 4].[8, 3, -6] = 0 M1* $d = \sqrt{3^2 + 0^2 + 4^2} = 5 \text{ M1(dep*) A1 A1}$ (maximum 5 / 6)			•
CP = [11, 3, -2] - [8, 3, -6] = ±[3, 0, 4] B1 Verify [3, 0, 4] . [8, 3, -6] = 0 M1* $d = \sqrt{3^2 + 0^2 + 4^2} = 5 M1(dep^*) A1 A1$ (maximum 5 / 6)	(a =) 5	A1 6	
Verify $[3, 0, 4] \cdot [8, 3, -6] = 0$ M1* $d = \sqrt{3^2 + 0^2 + 4^2} = 5 \text{M1(dep*) A1 A1} $ (maximum 5 / 6)			
$d = \sqrt{3^2 + 0^2 + 4^2} = 5 \text{M1(dep*) A1 A1} $ (maximum 5 / 6)			
(maximum 5 / 6)			
6			
		6	

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4 Integrating factor $e^{\int -\frac{x^2}{1+x^3}dx}$	M1	For correct process for finding integrating factor
$= e^{-\frac{1}{3}\ln(1+x^3)} = \left(1+x^3\right)^{-\frac{1}{3}}$	A1	For correct IF, simplified (here or later)
$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}x} \left(y \left(1 + x^3 \right)^{-\frac{1}{3}} \right) = \frac{x^2}{\left(1 + x^3 \right)^{\frac{1}{3}}}$	M1	For multiplying through by their IF
$\Rightarrow y(1+x^3)^{-\frac{1}{3}} = \frac{1}{2}(1+x^3)^{\frac{2}{3}} (+c)$	M1	For integrating RHS to obtain $A(1+x^3)^k$ OR $\ln A(1+x^3)^k$
	A1	For correct integration (+c not required here)
$\Rightarrow 1 = \frac{1}{2} + c \Rightarrow c = \frac{1}{2}$	M1 A1 √	For substituting (0, 1) into GS (including + c)
$\Rightarrow y = \frac{1}{2}(1+x^3) + \frac{1}{2}(1+x^3)^{\frac{1}{3}}$	A1	For correct <i>c</i> . f.t. from their GS For correct solution. AEF in form $y = f(x)$
$\Rightarrow y = \frac{1}{2}(1+x^2) + \frac{1}{2}(1+x^2)$	8	Tor correct solution. ALI in form $y = I(x)$
5 (i) EITHER $\mathbf{a} = [2, 3, 5], \mathbf{b} = \pm [2, 2, 0]$	B1	For stating 2 vectors in the plane
$\mathbf{n} = \mathbf{a} \times \mathbf{b} = \pm k [-10, 10, -2]$	M1 A1 √	For finding perpendicular to plane For correct n . f.t. from incorrect b
Use (2, 1, 5) OR (0, -1, 5)	M1	For substituting a point into equation $ax + by + cz = d$ where $[a, b, c]$ = their n
$\Rightarrow 5x - 5y + z = 10$	A1	For correct cartesian equation AEF
OR $\mathbf{a} = [2, 3, 5], \mathbf{b} = \pm [2, 2, 0]$	B1	For stating 2 vectors in the plane
e.g. $\mathbf{r} = [2, 1, 5] + \lambda[2, 2, 0] + \mu[2, 3, 5]$	M1	For stating parametric equation of plane
$[x, y, z] = [2 + 2\lambda + 2\mu, 1 + 2\lambda + 3\mu, 5 + 5\mu]$	A1 √	For writing 3 equations in <i>x</i> , <i>y</i> , <i>z</i> f.t. from incorrect b
	M1	For eliminating λ and μ
$\Rightarrow 5x - 5y + z = 10$	A1 5	For correct cartesian equation AEF
(ii) $[2t, 3t-4, 5t-9]$	B1 1	For stating a point A on l_1 with parameter t AEF
(iii) $\pm [2t+5, 3t-7, 5t-13]$	M1	For finding direction of l_2 from A and (–
$\pm [2t+5, 3t-7, 5t-13] \cdot [2, 3, 5] = 0$	M1	5,3, 4) For using scalar product for perpendicularity with any vector involving
$\Rightarrow t = 2$	A1	t For correct value of t
$\frac{x+5}{9} = \frac{y-3}{-1} = \frac{z-4}{-3}$ OR	A1 4	For a correct equation AEFcartesian
$\frac{x-4}{9} = \frac{y-2}{-1} = \frac{z-1}{-3}$		
9 -1 -3	10	SR For $2p+3q+5r=0$ and no further progress award B1

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6 (i) $(m^2 + 4 = 0 \Rightarrow) m = \pm 2i$	B1	For correct solutions of auxiliary equation (may be implied by correct CF)
$CF = A\cos 2x + B\sin 2x$	B1	For correct CF (AEtrig but not $Ae^{2ix} + Be^{-2ix}$ only)
$PI = p\sin x (+ q\cos x)$	B1	State a trial PI with at least $p \sin x$
$-p\sin x (-q\cos x) + 4p\sin x (+4q\cos x) = \sin x$	M1	For substituting PI into DE
$\Rightarrow p = \frac{1}{3}, q = 0$	A1	For correct <i>p</i> and <i>q</i> (which may be implied)
$\Rightarrow y = A\cos 2x + B\sin 2x + \frac{1}{3}\sin x$	B1 √ 6	For using GS = CF + PI, with 2 arbitrary constants in CF and none in PI
(ii) $(0,0) \Rightarrow A = 0$	B1 √	For correct equation in A and/or B f.t. from their GS
$\frac{\mathrm{d}y}{\mathrm{d}x} = 2B\cos 2x + \frac{1}{3}\cos x \Rightarrow \frac{4}{3} = 2B + \frac{1}{3}$	M1	For differentiating their GS and
dx 3 3 3		substituting values for x and $\frac{dy}{dx}$
$A = 0, \ B = \frac{1}{2}$	A1	For correct A and B
		Allow $A = -\frac{1}{4}i$, $B = \frac{1}{4}i$ from
		$CF A e^{2i x} + B e^{-2i x}$
$\Rightarrow y = \frac{1}{2}\sin 2x + \frac{1}{3}\sin x$	A1 4	For stating correct solution CAO
	10	
7 (i) $C + iS = 1 + e^{i\theta} + e^{2i\theta} + e^{3i\theta} + e^{4i\theta} + e^{5i\theta}$	M1	For using de Moivre, showing at least 3 terms
$e^{6i\theta}$ –1	M1	For recognising GP
$=\frac{e^{6i\theta}-1}{e^{i\theta}-1}$	A1	For correct GP sum
$= \frac{e^{3i\theta} - e^{-3i\theta}}{e^{\frac{1}{2}i\theta} - e^{-\frac{1}{2}i\theta}} \cdot \frac{e^{3i\theta}}{e^{\frac{1}{2}i\theta}} = \frac{e^{3i\theta} - e^{-3i\theta}}{e^{\frac{1}{2}i\theta} - e^{-\frac{1}{2}i\theta}} e^{\frac{5}{2}i\theta}$	A1 4	For obtaining correct expression AG
(ii) $C + iS = \frac{2i\sin 3\theta}{2i\sin \frac{1}{2}\theta} \cdot e^{\frac{5}{2}i\theta}$	M1	For expressing numerator and denominator in terms of sines
$21\sin\frac{1}{2}\theta$	A1	For $k \sin 3\theta$ and $k \sin \frac{1}{2}\theta$
$Re \Rightarrow C = \sin 3\theta \cos \frac{5}{2}\theta \csc \frac{1}{2}\theta$	A1	For correct expression AG
$Im \Rightarrow S = \sin 3\theta \sin \frac{5}{2}\theta \csc \frac{1}{2}\theta$	B1 4	For correct expression
(iii) $C = S \implies \sin 3\theta = 0, \tan \frac{5}{2}\theta = 1$	M1	For either equation deduced AEF
0 1 2		Ignore values outside $0 < \theta < \pi$
$\theta = \frac{1}{3}\pi, \frac{2}{3}\pi$	A1	For both values correct and no extras
$\theta = \frac{1}{10} \pi, \frac{1}{2} \pi, \frac{9}{10} \pi$	A2 4	For all values correct and no extras.
		Allow A1 for any 1 value <i>OR</i> all correct with extras
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8 (i) $r^4 . a \neq a . r^4$	B1 1	For stating the non-commutative product in the given table, or justifying another correct one
(ii) Possible subgroups order 2, 5	B1 B1 2	For either order stated For both orders stated, and no more (Ignore 1)
(iii) (a) $\{e, a\}$	B1	For correct subgroup
(b) $\{e, r, r^2, r^3, r^4\}$	B1 2	For correct subgroup
(iv) order of $r^3 = 5$	B1	For correct order
$(ar)^2 = ar.ar = r^4 a.ar = e$	M1	For attempt to find $(ar)^m = e$ OR
		$(ar^2)^m = e$
\Rightarrow order of $ar = 2$	A1	For correct order
$(ar^{2})^{2} = ar^{2}ar.r = ar^{2}r^{4}a.r = ara.r = e$		
\Rightarrow order of $ar^2 = 2$	A1 4	For correct order
(v) $ \frac{ ar ar^2 ar^3 ar^4 }{ ar e r r^2 r^3 } $ $ ar^2 r^4 e r r^2 $	B1	If the border elements $ar ar^2 ar^3 ar^4$ are not written, it will be assumed that the products arise from that order For all 16 elements of the form e or r^m
$\begin{vmatrix} ar^{2} & r^{4} & e & r & r^{2} \\ ar^{3} & r^{3} & r^{4} & e & r \\ ar^{4} & r^{2} & r^{3} & r^{4} & e \end{vmatrix}$	B1 B1	For all 4 elements in leading diagonal = e For no repeated elements in any completed row or column
	B1 B1 5	For any two rows or columns correct For all elements correct