## Mark Scheme 4733 June 2006

1		$x^4$ $\begin{bmatrix} x^4 \end{bmatrix}^4$ $\begin{bmatrix} x^4 \end{bmatrix}^4$	M1		Integrate xf(x), limits 3 & 4 [can be implied]
		$\mu = \frac{3}{37} \int_{3}^{4} x^{3} dx = \frac{3}{37} \left[ \frac{x^{4}}{4} \right]_{3}^{4} \left[ = 3 \frac{81}{148} \right]$			$\left[\frac{525}{148} \text{ or } 3.547\right]$
		$\begin{bmatrix} x^5 \end{bmatrix}^4$	M1		Attempt to integrate $x^2 f(x)$ , limits 3 & 4
		$\frac{3}{37}\int_{3}^{4} x^{4} dx = \frac{3}{37}\left[\frac{x^{5}}{5}\right]_{3}^{4}$	A1		Correct indefinite integral, any form
		$= 12\frac{123}{185} \text{ or } 12.665$	A1		$\frac{2343}{185}$ or in range [12.6, 12.7] [can be implied]
		$\sigma^2 = 12\frac{123}{185} - 3\frac{81}{148}^2 = 0.0815$	M1		Subtract their $\mu^2$
		100	A1	6	Answer, in range [0.0575, 0.084]
2	(i)	Find $P(R \ge 6)$ or $P(R < 6)$	M1		Find P(= 6) from tables/calc, OR RH critical
		= 0.0083 or 0.9917	A1		region P(≥ 6) in range [0.008, 0.0083] or P(< 6) =
		Compare with 0.025 [can be from	B1		$P(\ge 0)$ in range [0.006, 0.0063] of $P(< 0) = 0.9917$
		N]		4	OR CR is 6 with probability
		[0.05 if "empty LH tail	A1√		0.0083/0.9917
		stated]			Explicitly compare with 0.025 [or 0.975 if
		Reject H <sub>0</sub>			consistent] OR state that result is in critical region
					Correct comparison and conclusion, $$ on their $p$
	(ii)	$n = 9$ , $P(\le 1) = 0.0385$ [> 0.025]	M1		At least one, or $n = 8$ , $P(\le 1) = 0.0632$
	, ,	$n = 10, P(\le 1) = 0.0233 [< 0.025]$	A1		Both of these probabilities seen, don't need
		Therefore $n = 9$	B1	3	0.025
					Answer $n = 9$ only, indep't of M1A1, not from P(= 1)
3	(i)	$(140 - \mu)/\sigma = -2.326$	M1		One standardisation equated to $\Phi^{-1}$ , allow "1–",
	(-)	$(300 - \mu)/\sigma = 0.842$	B1		$\sigma^2$
		(	A1√		Both 2.33 and 0.84 at least, ignore signs
		Solve to obtain:	M1		Both equations completely correct, $$ on their $z$
		$\mu = 257.49$	A1 A1		Solve two simultaneous equations to find one
		$\sigma$ = <b>50.51</b>	Ai	6	variable
					μ value, in range [257, 258] σ in range [50.4, 50.55]
	(ii)	Higher	B1		"Higher" or equivalent stated
	` ,	as there is positive skew	B1	2	Plausible reason, allow from normal calculations
4	(i)	Each element equally likely to be	B1	1	One of these two. "Selections independent"
		selected (and all selections			alone is insufficient, but don't need this. An
		independent) OR each possible sample equally likely			example is insufficient.
	(ii)	B(6, 5/8)	M1		B(6, 5/8) stated or implied, allow e.g. 499/799
		${}^{6}C_{4} p^{4} (1-p)^{2}$	M1		Correct formula, any p
		= 0.32187	A1√	3	Answer, a.r.t. 0.322, can allow from wrong <i>p</i>
	(iii)	N(37.5, 225/16)	B1		Normal, mean 37.5, or 37.47 from 499/799,
		$\frac{39.5 - 37.5}{2.75} = 0.5333$	B1 M1 de	n	499/800 14.0625 or 3.75 seen, allow 14.07/14.1 or 3.75
		3.75	A1	۲	Standardise, wrong or no cc, $np$ , $npq$ , no $\sqrt{n}$
		$1 - \Phi(0.5333)$	dep M	1	Correct cc, $\sqrt{npq}$ , signs can be reversed
		= 0.297	A1		Tables used, answer $< 0.5$ , $p = 5/8$
			6		Answer, a.r.t. 0.297
					SR: $np < 5$ : Po( $np$ ) stated or implied,
	ļ		<u> </u>		B1

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AG			≈ Po(3.03)	B1 2	• •
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		(ii)		_	Correct formula, ± 1 term or "1 – " or both Convincingly obtain 0.4165(02542) [Exact:
$e^{-3.02} (1+3.02) = 0.1962 \qquad \qquad \text{M1} \qquad \qquad \text{Formula, at least one correct term} \\ 0.196 < 0.2 \qquad \qquad \text{A1} \qquad \qquad \text{O.1962 [or 0.1947 from exact]} \\ \text{So 302 seats.} \qquad \qquad \text{A1} \qquad \qquad \text{S} \qquad \text{Nower 302 only} \\ \text{SR:} \qquad \text{B (303, 0.99): B1B0; M0; M1 then N(298.98,2.9898) or equiv, standardise: M1A1 total 4/9} \\ \text{SR:} \qquad p = 0.1: \qquad \text{B (303, 0.1), N(30.3, 27.27) B1B0;} \qquad \text{Standardise 2 with } np \& \sqrt{npq}, \text{M1A0;} \\ \text{N(0.1}n, 0.09n); \text{ standardise with } np \& \sqrt{npq}; \text{ solve quadratic for } \sqrt{n}, n = 339: \qquad \text{M1M1M1A1, total M1A1} \\ \text{SR:} \qquad 6/9 \\ \text{B (303, 0.01)} \approx \text{N(3.03, 2.9997): B1B0; M0A0; M1A0} \\ \text{6} \qquad \text{(ii)} \qquad \text{Customers arrive independently} \qquad \text{B1} \qquad 1 \qquad \text{Valid reason in context, allow "random"} \\ \text{(iii)} \qquad 1 - 0.9921 \qquad \qquad \text{M1} \qquad Poisson tables, "1 -", or correct formula $\pm$ 1 term and $\pm$ 1 term and $\pm$ 2 and $\pm$ 2 and $\pm$ 3 and $\pm$ 3 and $\pm$ 4 and $\pm$ 3 and $\pm$ 4 and $$		/iii\	202 coato — – 2 02	N/1	Try amallar value of
A1		(111)			
0.196 < 0.2 So <b>302</b> seats.  SR: B(303, 0.99): B1B0; M0; M1 then N(298.98,2.9898) or equiv, standardise: M1A1 total 4/9 SR: $p = 0.1$ : B(303, 0.1), N(30.3, 27.27) B1B0; Standardise 2 with $np \& \sqrt{npq}$ , M1A0; N(0.1 $n$ , 0.09 $n$ ); standardise with $np \& \sqrt{npq}$ ; solve quadratic for $\sqrt{n}$ ; $n = 339$ : M1M1M1A1, to SR: 6/9 B(303, 0.01) ≈ N(3.03, 2.9997): B1B0; M0A0; M1A0 <b>6</b> (i) Customers arrive independently B1 1 Valid reason in context, allow "random"  (iii) 1 − 0.9921			$e^{-(1+3.02)} = 0.1302$		· ·
SR: B(303, 0.99): B1B0; M0; M1 then N(298.98,2.9898) or equiv, standardise: M1A1 total 4/9 SR: $p = 0.1$ : B(303, 0.1), N(30.3, 27.27) B1B0; Standardise 2 with $np \& \sqrt{npq}$ , M1A0; N(0.1 $n$ , 0.09 $n$ ); standardise with $np \& \sqrt{npq}$ ; solve quadratic for $\sqrt{n}$ ; $n = 339$ : M1M1M1A1, to SR: 6/9  B(303, 0.01) $\approx$ N(3.03, 2.9997): B1B0; M0A0; M1A0  6 (i) Customers arrive independently B1 1 Valid reason in context, allow "random"  (ii) $1 - 0.9921$ M1 Poisson tables, "1 –", or correct formula $\pm$ 1 term A1 2 Answer, a.r.t. 0.008 [1 – 0.9384 = 0.0606: M1.000]  (iii) N(48, 48) B1 Normal, mean 48 Variance or SD same as mean $\sqrt{npq}$ Standardise, wrong or no cc, $\sqrt{npq}$ A1 dep M1 Use tables, answer < 0.5  = 0.1394 A1 6 Answer in range [0.139, 0.14]  (iv) $e^{-\lambda} < 0.02$ M1 Correct formula for P(0), OR P(0   $\lambda = 4$ ) at least by T & I and the standardise of R \(\lambda = 3.912\) A1 3.91(2) seen OR $\lambda = 3.91$ at least by T & I			0.196 < 0.2		
SR: $p = 0.1$ : B(303, 0.1), N(30.3, 27.27) B1B0; Standardise 2 with $np \& \sqrt{npq}$ , M1A0; N(0.1 $n$ , 0.09 $n$ ); standardise with $np \& \sqrt{npq}$ ; solve quadratic for $\sqrt{n}$ ; $n = 339$ : M1M1M1A1, to SR: $6/9$ B(303, 0.01) $\approx$ N(3.03, 2.9997): B1B0; M0A0; M1A0  6 (i) Customers arrive independently B1 1 Valid reason in context, allow "random"  (ii) $1 - 0.9921$ $= 0.0079$ M1 Poisson tables, "1 –", or correct formula $\pm$ 1 term A1 2 Answer, a.r.t. 0.008 [1 – 0.9384 = 0.0606: M1.]  (iii) N(48, 48) $z = \frac{55.5 - 48}{\sqrt{48}}$ M1 dep Standardise, wrong or no cc, $\mu = \lambda$ Correct cc, $\sqrt{\lambda}$ 1 – $\Phi$ (1.0825) $= 0.1394$ A1 6 Answer in range [0.139, 0.14]  (iv) $e^{-\lambda} < 0.02$ M1 Correct formula for P(0), OR P(0   $\lambda = 4$ ) at least by T & I and the standardise of the point of the					
N(0.1 n, 0.09 n); standardise with np & √npq; solve quadratic for √n; n = 339: M1M1M1A1, to SR: 6/9					
SR: $6/9$ $B(303, 0.01) \approx N(3.03, 2.9997)$ : B1B0; M0A0; M1A0  6 (i) Customers arrive independently		SR:			
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		` ′	= 0.0079	A1 <b>2</b>	Answer, a.r.t. 0.008 [1 – 0.9384 = 0.0606: M1A0]
		(iii)	N(48, 48)		
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= 3.912 A1 3.91(2) seen OR $\lambda$ = 3.91 at least by T & I		(17)			` ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' '
1 - 1 - 0.912. $1 = 3.70$ Hilliukes $1 = 1.70$ H			0.4t = 3.912: $t = 9.78$ minutes	M1	Divide $\lambda$ by 0.4 or multiply by 150, any distribution
$t = 9$ minutes 47 seconds A1 5 587 seconds $\pm 1$ sec [inequalities not needed]				A1 5	

7	(i)	$\frac{c - 4000}{60 / \sqrt{50}} = 1.645$ Solve $c = 4014 \qquad [4013.958]$ Critical region is > <b>4014</b>	M1 B1 A1√ M1 A1 A1√ 6	Standardise unknown with $\sqrt{50}$ or $50$ [ignore RHS] $z = 1.645$ or $-1.645$ seen Wholly correct eqn, $$ on their $z$ [1 - 1.645: M1B1A0] Solve to find $c$ Value of $c$ , a.r.t. $4014$ Answer "> $4014$ ", allow $\geq$ , $$ on their $c$ , needs M1M1
	(ii)	Use "Type II is: accept when $H_0$ false" $\frac{4020 - 4014}{60 / \sqrt{50}} = 0.7071  [0.712 \text{ from} \\ 4013.958] \\ 1 - \Phi(0.7071) \\ = \textbf{0.240} \qquad [0.238 \text{ from} \\ 4013.958]$	M1dep depM1 A1√ A1 M1 A1 6	Standardise 4020 and $4014$ , allow $60^2$ , cc With $\sqrt{50}$ or 50 Completely correct LHS, $$ on their $c$ z-value in range [0.707, 0.712] Normal tables, answer < 0.5 Answer in range [0.2375, 0.2405]
	(iii)	Smaller Smaller cv, better test etc	B1 B1 <b>2</b>	"Smaller" stated, no invalidating reason Plausible reason
	(iv)	Smaller Smaller cv, larger prob of Type I etc	B1 B1 <b>2</b>	"Smaller" stated, no invalidating reason Plausible reason
	(v)	No, parent distribution known to be normal	B2 <b>2</b>	"No" stated, convincing reason SR: If B0, "No", reason that is not invalidating: B1