Mark Scheme 4733 June 2006

| 1 | $\begin{aligned} & \mu=\frac{3}{37} \int_{3}^{4} x^{3} d x=\frac{3}{37}\left[\frac{x^{4}}{4}\right]_{3}^{4}\left[=3 \frac{81}{148}\right] \\ & \frac{3}{37} \int_{3}^{4} x^{4} d x=\frac{3}{37}\left[\frac{x^{5}}{5}\right]_{3}^{4} \\ & =12 \frac{123}{185} \text { or } 12.665 \\ & \sigma^{2}=12 \frac{123}{185}-3 \frac{81}{148}=\mathbf{0 . 0 8 1 5} \end{aligned}$ | $\begin{array}{ll}\text { M1 } & \\ & \\ \text { M1 } & \\ \text { A1 } & \\ \text { A1 } & \\ \text { M1 } & \\ \text { A1 } & 6 \\ & \\ & \end{array}$ | ```Integrate \(\mathrm{xf}(x)\), limits 3 \& \(4 \quad\) [can be implied] [ \(\frac{525}{148}\) or 3.547] Attempt to integrate \(x^{2} \mathrm{f}(x)\), limits 3 \& 4 Correct indefinite integral, any form \(\frac{2343}{185}\) or in range [12.6, 12.7] [can be implied] Subtract their \(\mu^{2}\) Answer, in range [ \(0.0575,0.084\) ]``` |
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| 2 | (i) $\begin{array}{cl}\text { Find } \mathrm{P}(R \geq 6) & \text { or } \mathrm{P}(R<6) \\ =0.0083 & \text { or } 0.9917\end{array}$ <br> Compare with 0.025 [can be from <br> $\mathrm{N}]$ <br> [0.05 if "empty LH tail <br> stated] <br> Reject $\mathrm{H}_{0}$ | M1   <br> A1   <br> B1   <br> A1 $\sqrt{ }$ 4  | Find $\mathrm{P}(=6)$ from tables/calc, OR RH critical region <br> $P(\geq 6)$ in range $[0.008,0.0083]$ or $P(<6)=$ 0.9917 <br> OR CR is 6 with probability 0.0083/0.9917 <br> Explicitly compare with 0.025 [or 0.975 if consistent] <br> OR state that result is in critical region <br> Correct comparison and conclusion, $\sqrt{ }$ on their $p$ |
|  | (ii) $n=9, \mathrm{P}(\leq 1)=0.0385[>0.025]$ <br> $n=10, \mathrm{P}(\leq 1)=0.0233[<0.025]$ <br> Therefore $n=\mathbf{9}$ | $\begin{array}{ll} \mathrm{M1} & \\ \mathrm{~A} 1 & \\ \mathrm{~B} 1 & 3 \end{array}$ | At least one, or $n=8, P(\leq 1)=0.0632$ <br> Both of these probabilities seen, don't need 0.025 <br> Answer $n=9$ only, indep't of M1A1, not from P(= <br> 1) |
| 3 | (i) $\begin{aligned} & (140-\mu) / \sigma=-2.326 \\ & (300-\mu) / \sigma=0.842\end{aligned}$ <br> Solve to obtain: $\begin{aligned} \mu & =257.49 \\ \sigma & =50.51 \end{aligned}$ |  M1  <br> B1   <br> A1 $\sqrt{2}$   <br> M1   <br> A1   <br> A1 6  | One standardisation equated to $\Phi^{-1}$, allow " 1 -", $\sigma^{2}$ <br> Both 2.33 and 0.84 at least, ignore signs Both equations completely correct, $\sqrt{ }$ on their $z$ Solve two simultaneous equations to find one variable <br> $\mu$ value, in range [257, 258] <br> $\sigma$ in range [50.4, 50.55] |
|  | (ii) Higher as there is positive skew | $\begin{array}{ll}  \\ \hline \mathrm{Bi} & \\ \mathrm{~B} 1 & \mathbf{2} \\ \hline \end{array}$ | "Higher" or equivalent stated Plausible reason, allow from normal calculations |
| 4 | (i) Each element equally likely to be selected (and all selections independent) OR each possible sample equally likely | B1 | One of these two. "Selections independent" alone is insufficient, but don't need this. An example is insufficient. |
|  | $\text { (ii) } \begin{aligned} & \quad \begin{array}{l} B(6,5 / 8) \\ \\ \\ { }^{6} C_{4} p^{4}(1-p)^{2} \\ =0.32187 \end{array} \end{aligned}$ | M1 <br> M1 <br> A1 3 | $\mathrm{B}(6,5 / 8)$ stated or implied, allow e.g. 499/799 Correct formula, any $p$ Answer, a.r.t. 0.322, can allow from wrong $p$ |
|  | $\text { (iii) } \begin{aligned} & \mathrm{N}(37.5,225 / 16) \\ & \frac{39.5-37.5}{3.75}=0.5333 \\ & \\ & \\ & \\ & =\mathbf{0 . 2 9 7} \end{aligned}$ | B1 <br> M1 dep <br> A1 <br> dep M1 <br> A1 <br> 6 | Normal, mean 37.5, or 37.47 from 499/799, 499/800 <br> 14.0625 or 3.75 seen, allow $14.07 / 14.1$ or 3.75 Standardise, wrong or no cc, np, npq, no $\sqrt{ } n$ Correct cc, $\sqrt{ } n p q$, signs can be reversed Tables used, answer < 0.5, $p=5 / 8$ Answer, a.r.t. 0.297 <br> SR: $n p<5$ : $\quad \operatorname{Po}(n p)$ stated or implied, <br> B1 |


| 5 | (i) | $\begin{aligned} & \mathrm{B}(303,0.01) \\ & \approx \mathrm{Po}(3.03) \end{aligned}$ | B1 <br> B1 2 | $\mathrm{B}(303,0.01)$ stated, allow $p=0.99$ or 0.1 <br> Allow Bin implied clearly by parameters <br> $\mathrm{Po}(3.03)$ stated or implied, can be recovered from <br> (ii) |
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|  | (ii) | $e^{-3.03}\left(1+3.03+\frac{3.03^{2}}{2}\right)=0.4165$ <br> AG | $\begin{array}{ll}  & \\ \text { M1 } & 2 \end{array}$ | Correct formula, $\pm 1$ term or "1 - " or both Convincingly obtain $0.4165(02542)$ [Exact: $0.41535]$ |
|  | (iii) | $\begin{aligned} & 302 \text { seats } \Rightarrow \mu=3.02 \\ & e^{-3.02}(1+3.02)=0.1962 \\ & 0.196<0.2 \\ & \text { So } 302 \text { seats. } \end{aligned}$ | M1 <br> M1 <br> A1 <br> A1 <br> A1 <br> 5 | Try smaller value of $\mu$ Formula, at least one correct term Correct number of terms for their $\mu$ 0.1962 [or 0.1947 from exact] Answer 302 only |
|  | $\begin{aligned} & \text { SR: } \\ & \text { SR: } \\ & \text { SR: } \end{aligned}$ | $\begin{aligned} & \begin{array}{l} \mathrm{B}(303,0.99): \\ p=0.1: r \\ \mathrm{~N} 1 \mathrm{~B} 0 ; \mathrm{M} 0 ; \mathrm{M} 1 \text { then } \\ \mathrm{N}(0.1 n, 0.09 n) ; \text { standardise } \mathrm{n} \\ 6 / 9 \\ \mathrm{~B}(303,0.01) \approx \mathrm{N}(3.03,2.9997): \mathrm{B} \end{array} \end{aligned}$ | $\begin{aligned} & 298.98,2.98 \\ & 27.27) \mathrm{B} 1 \mathrm{~B} \\ & n p \& \text { Vnpq } \\ & 0 ; \text { MOAO; } \end{aligned}$ | 88) or equiv, standardise: M1A1 total 4/9 <br> Standardise 2 with $n p$ \& $\sqrt{n p q}$, M1A0; solve quadratic for $\sqrt{ } n ; n=339$ : M1M1M1A1, total 1A0 |
| 6 | (i) | Customers arrive independently | B1 1 | Valid reason in context, allow "random" |
|  | (ii) | $\begin{aligned} & 1-0.9921 \\ & =0.0079 \end{aligned}$ | $\begin{array}{ll} \mathrm{M} 1 & \\ \text { A1 } & 2 \end{array}$ | Poisson tables, " $1-$ ", or correct formula $\pm 1$ term Answer, a.r.t. $0.008 \quad[1-0.9384=0.0606$ : M1A0 $]$ |
|  | (iii) | $\begin{aligned} & \begin{array}{l} N(48,48) \\ z=\frac{55.5-48}{\sqrt{48}} \\ =1.0825 \end{array} \\ & 1-\Phi(1.0825) \\ & =0.1394 \end{aligned}$ | B1 <br> B1 $\sqrt{ }$ <br> M1 dep <br> A1 <br> dep M1 <br> A1 <br> 6 | Normal, mean 48 <br> Variance or SD same as mean $\sqrt{ }$ <br> Standardise, wrong or no cc, $\mu=\lambda$ <br> Correct cc, $\sqrt{\lambda}$ <br> Use tables, answer < 0.5 <br> Answer in range [0.139, 0.14] |
|  | (iv) | $\begin{aligned} & e^{-\lambda}<0.02 \\ & \lambda>-\ln 0.02 \\ & \quad=3.912 \\ & 0.4 t=3.912: \quad t=9.78 \text { minutes } \\ & t=9 \text { minutes } 47 \text { seconds } \end{aligned}$ | M1 <br> M1 <br> A1 <br> M1 <br> A1 $5$ | Correct formula for $\mathrm{P}(0)$, OR $\mathrm{P}(0 \mid \lambda=4)$ at least In used $\quad$ OR $\lambda=3.9$ at least by $T$ \& I 3.91(2) seen OR $\lambda=3.91$ at least by $T \& I$ Divide $\lambda$ by 0.4 or multiply by 150, any distribution 587 seconds $\pm 1 \mathrm{sec}$ [inequalities not needed] |


| 7 |  | $\frac{c-4000}{60 / \sqrt{50}}=1.645$ <br> Solve $c=4014 \quad \text { [4013.958] }$ <br> Critical region is $\mathbf{>} 4014$ | M1 <br> B1 <br> A1 $\sqrt{ }$ <br> M1 <br> A1 <br> A1 $\sqrt{ }$ <br> 6 | Standardise unknown with $\sqrt{ } 50$ or $50 \quad$ [ignore RHS] $z=1.645 \text { or }-1.645 \text { seen }$ <br> Wholly correct eqn, $\sqrt{ }$ on their $z[1-1.645$ : <br> M1B1A0] <br> Solve to find $c$ <br> Value of $c$, a.r.t. 4014 <br> Answer " $>4014$ ", allow $\geq, \sqrt{ }$ on their $c$, needs <br> M1M1 |
| :---: | :---: | :---: | :---: | :---: |
|  | (ii) | Use "Type II is: accept when $\mathrm{H}_{0}$ false" $\begin{array}{cc} \begin{array}{c} 4020-4014 \\ \hline 60 / \sqrt{50} \end{array} & \\ =0.7071 & \\ 4013.958] & \\ 1-\Phi(0.712 \text { from } \\ =0.240 & \\ 4013.958] & \\ \hline 0.238 \text { from } \end{array}$ | M1dep depM1 A1 $\sqrt{ }$ <br> A1 <br> M1 <br> A1 | Standardise 4020 and $4014 \sqrt{ }$, allow $60^{2}$, cc With $\sqrt{ } 50$ or 50 <br> Completely correct LHS, $\sqrt{ }$ on their $c$ $z$-value in range [0.707, 0.712] <br> Normal tables, answer < 0.5 <br> Answer in range [0.2375, 0.2405] |
|  | (iii) | Smaller <br> Smaller cv, better test etc | $\begin{array}{ll} B 1 \\ B 1 \end{array}$ | "Smaller" stated, no invalidating reason Plausible reason |
|  | (iv) | Smaller <br> Smaller cv, larger prob of Type I etc | $\begin{array}{ll}  \\ \text { B1 } & \\ \text { B1 } \end{array}$ | "Smaller" stated, no invalidating reason Plausible reason |
|  | (v) | No, parent distribution known to be normal | B2 | "No" stated, convincing reason SR: If B0, "No", reason that is not invalidating: B1 |

