OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4754(A)

Applications of Advanced Mathematics (C4)

Paper A

Monday **12 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

NOTE

- This paper will be followed by **Paper B: Comprehension.**

This question paper consists of 5 printed pages and 3 blank pages.

Section A (36 marks)

1 Fig. 1 shows part of the graph of $y = \sin x - \sqrt{3}\cos x$.

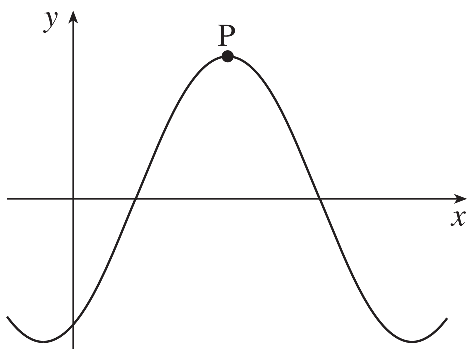

Fig. 1

Express $\sin x - \sqrt{3}\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 \leqslant \alpha \leqslant \frac{1}{2}\pi$.

Hence write down the exact coordinates of the turning point P. [6]

2 **(i)** Given that

$$\frac{3 + 2x^2}{(1 + x)^2(1 - 4x)} = \frac{A}{1 + x} + \frac{B}{(1 + x)^2} + \frac{C}{1 - 4x},$$

where A, B and C are constants, find B and C, and show that $A = 0$. [4]

(ii) Given that x is sufficiently small, find the first three terms of the binomial expansions of $(1 + x)^{-2}$ and $(1 - 4x)^{-1}$.

Hence find the first three terms of the expansion of $\dfrac{3 + 2x^2}{(1 + x)^2(1 - 4x)}$. [4]

3 Given that $\sin(\theta + \alpha) = 2\sin\theta$, show that $\tan\theta = \dfrac{\sin\alpha}{2 - \cos\alpha}$.

Hence solve the equation $\sin(\theta + 40°) = 2\sin\theta$, for $0° \leqslant \theta \leqslant 360°$. [7]

4 **(a)** The number of bacteria in a colony is increasing at a rate that is proportional to the square root of the number of bacteria present. Form a differential equation relating x, the number of bacteria, to the time t. [2]

(b) In another colony, the number of bacteria, y, after time t minutes is modelled by the differential equation

$$\frac{dy}{dt} = \frac{10000}{\sqrt{y}}.$$

Find y in terms of t, given that $y = 900$ when $t = 0$. Hence find the number of bacteria after 10 minutes. [6]

5 **(i)** Show that $\int x e^{-2x}\, dx = -\frac{1}{4} e^{-2x}(1 + 2x) + c$. [3]

A vase is made in the shape of the volume of revolution of the curve $y = x^{\frac{1}{2}} e^{-x}$ about the x-axis between $x = 0$ and $x = 2$ (see Fig. 5).

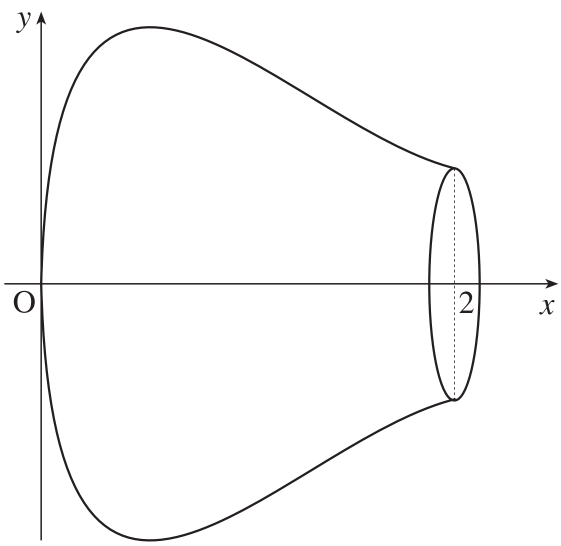

Fig. 5

(ii) Show that this volume of revolution is $\frac{1}{4}\pi\left(1 - \frac{5}{e^4}\right)$. [4]

[Turn over

Section B (36 marks)

6 Fig. 6 shows the arch ABCD of a bridge.

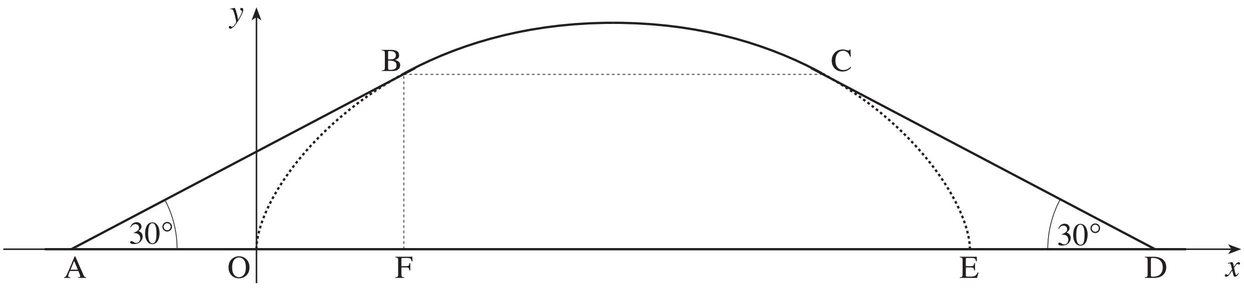

Fig. 6

The section from B to C is part of the curve OBCE with parametric equations

$$x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta) \text{ for } 0 \leqslant \theta \leqslant 2\pi,$$

where a is a constant.

(i) Find, in terms of a,

(A) the length of the straight line OE,

(B) the maximum height of the arch. [4]

(ii) Find $\dfrac{dy}{dx}$ in terms of θ. [3]

The straight line sections AB and CD are inclined at 30° to the horizontal, and are tangents to the curve at B and C respectively. BC is parallel to the x-axis. BF is parallel to the y-axis.

(iii) Show that at the point B the parameter θ satisfies the equation

$$\sin\theta = \frac{1}{\sqrt{3}}(1 - \cos\theta).$$

Verify that $\theta = \frac{2}{3}\pi$ is a solution of this equation.

Hence show that BF $= \frac{3}{2}a$, and find OF in terms of a, giving your answer exactly. [6]

(iv) Find BC and AF in terms of a.

Given that the straight line distance AD is 20 metres, calculate the value of a. [5]

7

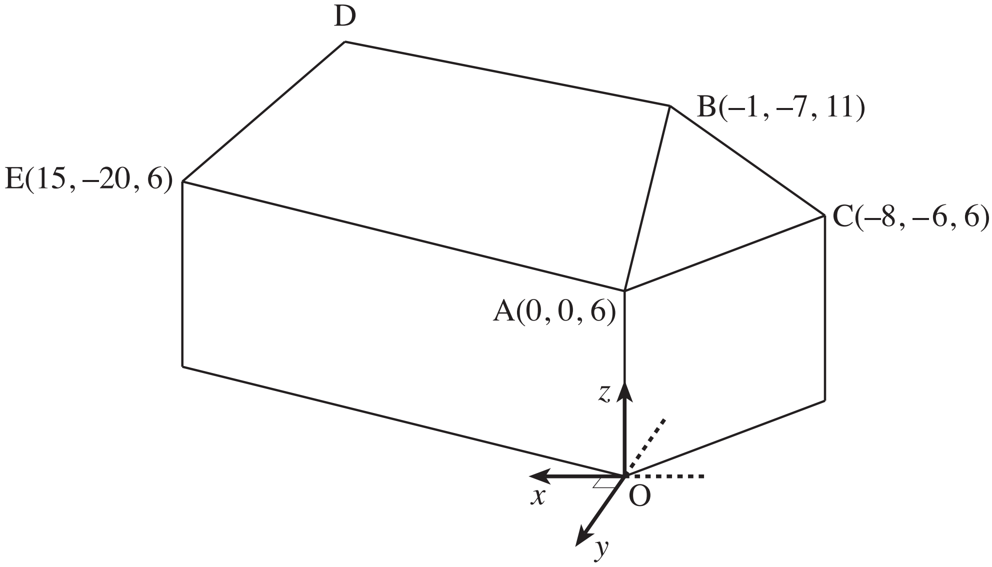

Fig. 7

Fig. 7 illustrates a house. All units are in metres. The coordinates of A, B, C and E are as shown. BD is horizontal and parallel to AE.

(i) Find the length AE. [2]

(ii) Find a vector equation of the line BD. Given that the length of BD is 15 metres, find the coordinates of D. [4]

(iii) Verify that the equation of the plane ABC is

$$-3x + 4y + 5z = 30.$$

Write down a vector normal to this plane. [4]

(iv) Show that the vector $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$ is normal to the plane ABDE. Hence find the equation of the plane ABDE. [4]

(v) Find the angle between the planes ABC and ABDE. [4]

Candidate Name	Centre Number	Candidate Number

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
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MEI STRUCTURED MATHEMATICS 4754(B)

Applications of Advanced Mathematics (C4)

Paper B: Comprehension

Monday **12 JUNE 2006** Afternoon Up to 1 hour

Additional materials:
Rough paper
MEI Examination Formulae and Tables (MF2)

TIME Up to 1 hour

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces at the top of this page.
- Answer **all** the questions.
- Write your answers in the spaces provided on the question paper.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 18.

For Examiner's Use	
Qu.	Mark
1	
2	
3	
4	
5	
6	
Total	

This question paper consists of 4 printed pages and an insert.

[Turn over

For Examiner's Use

1 The marathon is 26 miles and 385 yards long (1 mile is 1760 yards). There are now several men who can run 2 miles in 8 minutes. Imagine that an athlete maintains this average speed for a whole marathon. How long does the athlete take? [2]

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2 According to the linear model, in which calendar year would the record for the men's mile first become negative? [3]

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3 Explain the statement in line 93 "According to this model the 2-hour marathon will never be run." [1]

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For Examiner's Use

4 Explain how the equation in line 49,

$$R = L + (U - L)e^{-kt},$$

is consistent with Fig. 2

(i) initially, [3]

(ii) for large values of t. [2]

(i) ....

....

....

(ii) ....

....

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[Questions 5 and 6 are printed overleaf.]

For Examiner's Use

5 A model for an athletics record has the form

$$R = A - (A - B)\mathrm{e}^{-kt} \text{ where } A > B > 0 \text{ and } k > 0.$$

(i) Sketch the graph of R against t, showing A and B on your graph. [3]

(ii) Name one event for which this might be an appropriate model. [1]

(i)

(ii) ....

6 A number of cases of the general exponential model for the marathon are given in Table 6. One of these is

$$R = 115 + (175 - 115)\mathrm{e}^{-0.0467t^{0.797}}.$$

(i) What is the value of t for the year 2012? [1]

(ii) What record time does this model predict for the year 2012? [2]

(i) ....

(ii) ....

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