

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education**

MATHEMATICS

4726

Further Pure Mathematics 2

Tuesday

6 JUNE 2006

Afternoon

1 hour 30 minutes

Additional materials:

8 page answer booklet

Graph paper

List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

- 1** Find the first three non-zero terms of the Maclaurin series for

$$(1+x)\sin x,$$

simplifying the coefficients.

[3]

- 2** (i) Given that $y = \tan^{-1} x$, prove that $\frac{dy}{dx} = \frac{1}{1+x^2}$.

[3]

(ii) Verify that $y = \tan^{-1} x$ satisfies the equation

$$(1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0.$$

[3]

- 3** The equation of a curve is $y = \frac{x+1}{x^2+3}$.

(i) State the equation of the asymptote of the curve.

[1]

(ii) Show that $-\frac{1}{6} \leq y \leq \frac{1}{2}$.

[5]

- 4** (i) Using the definition of $\cosh x$ in terms of e^x and e^{-x} , prove that

$$\cosh 2x = 2 \cosh^2 x - 1.$$

[3]

(ii) Hence solve the equation

$$\cosh 2x - 7 \cosh x = 3,$$

giving your answer in logarithmic form.

[4]

- 5** (i) Express $t^2 + t + 1$ in the form $(t+a)^2 + b$.

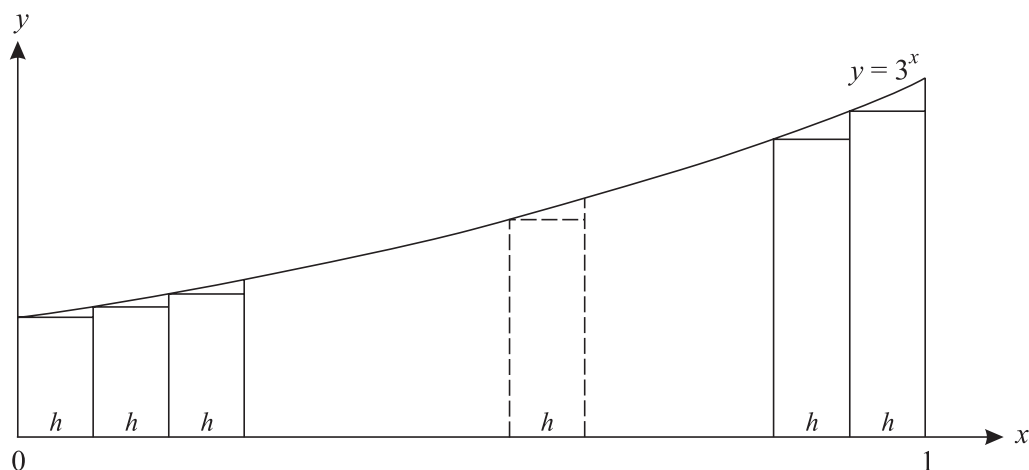
[1]

(ii) By using the substitution $\tan \frac{1}{2}x = t$, show that

$$\int_0^{\frac{1}{2}\pi} \frac{1}{2 + \sin x} dx = \frac{\sqrt{3}}{9} \pi.$$

[6]

6



The diagram shows the curve with equation $y = 3^x$ for $0 \leq x \leq 1$. The area A under the curve between these limits is divided into n strips, each of width h where $nh = 1$.

(i) By using the set of rectangles indicated on the diagram, show that $A > \frac{2h}{3^h - 1}$. [3]

(ii) By considering another set of rectangles, show that $A < \frac{(2h)3^h}{3^h - 1}$. [3]

(iii) Given that $h = 0.001$, use these inequalities to find values between which A lies. [2]

7 The equation of a curve, in polar coordinates, is

$$r = \sqrt{3} + \tan \theta, \quad \text{for } -\frac{1}{3}\pi \leq \theta \leq \frac{1}{4}\pi.$$

(i) Find the equation of the tangent at the pole. [2]

(ii) State the greatest value of r and the corresponding value of θ . [2]

(iii) Sketch the curve. [2]

(iv) Find the exact area of the region enclosed by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{4}\pi$. [5]

8 The curve with equation $y = \frac{\sinh x}{x^2}$, for $x > 0$, has one turning point.

(i) Show that the x -coordinate of the turning point satisfies the equation $x - 2 \tanh x = 0$. [3]

(ii) Use the Newton-Raphson method, with a first approximation $x_1 = 2$, to find the next two approximations, x_2 and x_3 , to the positive root of $x - 2 \tanh x = 0$. [5]

(iii) By considering the approximate errors in x_1 and x_2 , estimate the error in x_3 . (You are not expected to evaluate x_4 .) [3]

[Question 9 is printed overleaf.]

9 (i) Given that $y = \sinh^{-1} x$, prove that $y = \ln(x + \sqrt{x^2 + 1})$. [3]

(ii) It is given that, for non-negative integers n ,

$$I_n = \int_0^\alpha \sinh^n \theta \, d\theta,$$

where $\alpha = \sinh^{-1} 1$. Show that

$$nI_n = \sqrt{2} - (n-1)I_{n-2}, \quad \text{for } n \geq 2. \quad [6]$$

(iii) Evaluate I_4 , giving your answer in terms of $\sqrt{2}$ and logarithms. [4]