

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

MATHEMATICS 4730

Mechanics 3

Monday 22 MAY 2006 Morning 1 hour 30 minutes

Additional materials: 8 page answer booklet Graph paper List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer all the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g \, \text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.
- You are permitted to use a graphical calculator in this paper.

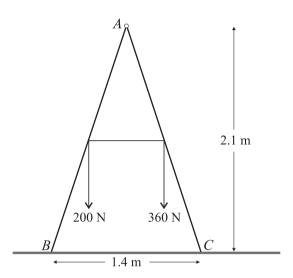
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- You are reminded of the need for clear presentation in your answers.

A ball of mass $0.4 \,\mathrm{kg}$ is moving in a straight line, with speed $25 \,\mathrm{m \, s^{-1}}$, when it is struck by a bat. The bat exerts an impulse of magnitude $20 \,\mathrm{N} \,\mathrm{s}$ and the ball is deflected through an angle of 90° . Calculate

- (i) the direction of the impulse, [3]
- (ii) the speed of the ball immediately after it is struck. [3]
- A duck of mass 2 kg is travelling with horizontal speed $4 \,\mathrm{m\,s^{-1}}$ when it lands on a lake. The duck is brought to rest by the action of resistive forces, acting in the direction opposite to the duck's motion and having total magnitude $(2v + 3v^2) \,\mathrm{N}$, where $v \,\mathrm{m\,s^{-1}}$ is the speed of the duck. Show that the duck comes to rest after travelling approximately 1.30 m from the point of its initial contact with the surface of the lake.

3

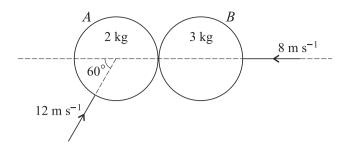


Two uniform rods AB and AC, of equal lengths, and of weights 200 N and 360 N respectively, are freely jointed at A. The mid-points of the rods are joined by a taut light inextensible string. The rods are in equilibrium in a vertical plane with B and C in contact with a smooth horizontal surface. The point A is 2.1 m above the surface and BC = 1.4 m (see diagram).

- (i) Show that the force exerted on AB at B has magnitude 240 N and find the tension in the string.
- (ii) Find the horizontal and vertical components of the force exerted on AB at A and state their directions. [3]
- 4 A particle is connected to a fixed point by a light inextensible string of length 2.45 m to make a simple pendulum. The particle is released from rest with the string taut and inclined at 0.1 radians to the downward vertical.
 - (i) Show that the motion of the particle is approximately simple harmonic with period 3.14 s, correct to 3 significant figures. [5]

Calculate, in either order,

- (ii) the angular speed of the pendulum when it has moved 0.04 radians from the initial position, [3]
- (iii) the time taken by the pendulum to move 0.04 radians from the initial position. [3]



Two uniform smooth spheres A and B, of equal radius, have masses 2 kg and 3 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision A is moving with speed $12 \,\mathrm{m\,s^{-1}}$ at 60° to the line of centres, and B is moving with speed $8 \,\mathrm{m\,s^{-1}}$ along the line of centres (see diagram). The coefficient of restitution between the spheres is 0.5. Find the speed and direction of motion of each sphere after the collision.

- 6 A bungee jumper of mass 70 kg is joined to a fixed point O by a light elastic rope of natural length 30 m and modulus of elasticity 1470 N. The jumper starts from rest at O and falls vertically. The jumper is modelled as a particle and air resistance is ignored.
 - (i) Find the distance fallen by the jumper when maximum speed is reached. [4]
 - (ii) Show that this maximum speed is $26.9 \,\mathrm{m \, s}^{-1}$, correct to 3 significant figures. [4]
 - (iii) Find the extension of the rope when the jumper is at the lowest position. [4]

[Question 7 is printed overleaf.]

4730/S06 **Turn over**

7

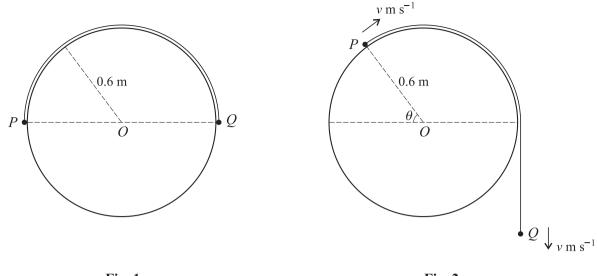


Fig. 1 Fig. 2

A smooth horizontal cylinder of radius $0.6\,\mathrm{m}$ is fixed with its axis horizontal and passing through a fixed point O. A light inextensible string of length $0.6\pi\,\mathrm{m}$ has particles P and Q, of masses $0.3\,\mathrm{kg}$ and $0.4\,\mathrm{kg}$ respectively, attached at its ends. The string passes over the cylinder and is held at rest with P, O and Q in a straight horizontal line (see Fig. 1). The string is released and Q begins to descend. When the line OP makes an angle θ radians, $0 \le \theta \le \frac{1}{2}\pi$, with the horizontal, the particles have speed $v\,\mathrm{m\,s}^{-1}$ (see Fig. 2).

(i) By considering the total energy of the system, or otherwise, show that

$$v^2 = 6.72\theta - 5.04\sin\theta.$$
 [5]

(ii) Show that the magnitude of the contact force between P and the cylinder is

$$(5.46 \sin \theta - 3.36\theta)$$
 newtons.

Hence find the value of θ for which the magnitude of the contact force is greatest. [6]

(iii) Find the transverse component of the acceleration of P in terms of θ . [3]