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Section A

1 $\sin \theta - 3 \cos \theta = R \sin(\theta - \alpha)$ $= R(\sin \theta \cos \alpha - \cos \theta \sin \alpha)$ $\Rightarrow R \cos \alpha = 1, R \sin \alpha = 3$ $\Rightarrow R^2 = 1^2 + 3^2 = 10 \Rightarrow R = \sqrt{10}$ $\tan \alpha = 3 \Rightarrow \alpha = 71.57^\circ$ $\sqrt{10} \sin(\theta - 71.57^\circ) = 1$ $\Rightarrow \theta - 71.57^\circ = \sin^{-1}(1/\sqrt{10})$ $\theta - 71.57^\circ = 18.43^\circ, 161.57^\circ$ $\Rightarrow \theta = 90^\circ, 233.1^\circ$	M1 B1 M1 A1 M1 B1 A1 [7]	equating correct pairs oe ft www cao (71.6° or better) oe ft R, α www and no others in range (MR-1 for radians)
2 Normal vectors are $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ $\Rightarrow \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = 2 - 6 + 4 = 0$ $\Rightarrow$ planes are perpendicular.	B1 B1 M1 E1 [4]	
3 (i) $y = \ln x \Rightarrow x = e^y$ $\Rightarrow V = \int_0^2 \pi x^2 dy$ $= \int_0^2 \pi (e^y)^2 dy = \int_0^2 \pi e^{2y} dy *$	B1 M1 E1 [3]	
(ii) $\int_0^2 \pi e^{2y} dy = \pi \left[\frac{1}{2} e^{2y} \right]_0^2$ $= \frac{1}{2} \pi (e^4 - 1)$	B1 M1 A1 [3]	$\frac{1}{2} e^{2y}$ substituting limits in $k\pi e^{2y}$ or equivalent, but must be exact and evaluate e^0 as 1.
4 $x = \frac{1}{t} - 1 \Rightarrow \frac{1}{t} = x + 1$ $\Rightarrow t = \frac{1}{x+1}$ $\Rightarrow y = \frac{2 + \frac{1}{x+1}}{1 + \frac{1}{x+1}} = \frac{2x+2+1}{x+1+1} = \frac{2x+3}{x+2}$	M1 A1 M1 E1	Solving for t in terms of x or y Subst their t which must include a fraction, clearing subsidiary fractions/ changing the subject oe www
or $\frac{3+2x}{2+x} = \frac{3 + \frac{2-2t}{t}}{2 + \frac{1-t}{t}}$ $= \frac{3t+2-2t}{2t+1-t}$ $= \frac{t+2}{t+1} = y$	M1 A1 M1 E1 [4]	substituting for x or y in terms of t clearing subsidiary fractions/ changing the subject

5 $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1-\lambda \\ 2+2\lambda \\ -1+3\lambda \end{pmatrix}$ When $x = -1$, $1 - \lambda = -1$, $\Rightarrow \lambda = 2$ $\Rightarrow y = 2 + 2\lambda = 6$, $z = -1 + 3\lambda = 5$ $\Rightarrow$ point lies on first line $\mathbf{r} = \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \mu \\ 6 \\ 3-2\mu \end{pmatrix}$ When $x = -1$, $\mu = -1$, $\Rightarrow y = 6$, $z = 3 - 2\mu = 5$ $\Rightarrow$ point lies on second line Angle between $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ is θ, where $\cos \theta = \frac{-1 \times 1 + 2 \times 0 + 3 \times -2}{\sqrt{14} \cdot \sqrt{5}}$ $= -\frac{7}{\sqrt{70}}$ $\Rightarrow \theta = 146.8^\circ$ $\Rightarrow$ acute angle is 33.2°	M1 E1 E1 M1 M1 A1 A1cao [7]	Finding λ or μ checking other two coordinates checking other two co-ordinates Finding angle between correct vectors use of formula $\pm \frac{7}{\sqrt{70}}$ Final answer must be acute angle
6(i) $A \approx 0.5 \left[\frac{(1.1696 + 1.0655)}{2} + 1.1060 \right]$ $= 1.11$ (3 s.f.)	M1 A1 cao [2]	Correct expression for trapezium rule
(ii) $(1 + e^{-x})^{1/2} = 1 + \frac{1}{2}e^{-x} + \frac{\frac{1}{2} \cdot -\frac{1}{2}}{2!}(e^{-x})^2 + \dots$ $\approx 1 + \frac{1}{2}e^{-x} - \frac{1}{8}e^{-2x} *$	M1 A1 E1 [3]	Binomial expansion with $p = \frac{1}{2}$ Correct coeffs
(iii) $I = \int_1^2 \left(1 + \frac{1}{2}e^{-x} - \frac{1}{8}e^{-2x} \right) dx$ $= \left[x - \frac{1}{2}e^{-x} + \frac{1}{16}e^{-2x} \right]_1^2$ $= \left(2 - \frac{1}{2}e^{-2} + \frac{1}{16}e^{-4} \right) - \left(1 - \frac{1}{2}e^{-1} + \frac{1}{16}e^{-2} \right)$ $= 1.9335 - 0.8245$ $= 1.11$ (3 s.f.)	M1 A1 A1 [3]	integration substituting limits into correct expression

Section B

7 (a) (i) $P_{\max} = \frac{2}{2-1} = 2$ $P_{\min} = \frac{2}{2+1} = 2/3.$	B1 B1 [2]	
(ii) $P = \frac{2}{2-\sin t} = 2(2-\sin t)^{-1}$ $\Rightarrow \frac{dP}{dt} = -2(2-\sin t)^{-2} \cdot -\cos t$ $= \frac{2\cos t}{(2-\sin t)^2}$ $\frac{1}{2}P^2 \cos t = \frac{1}{2} \frac{4}{(2-\sin t)^2} \cos t$ $= \frac{2\cos t}{(2-\sin t)^2} = \frac{dP}{dt}$	M1 B1 A1 DM1 E1 [5]	chain rule $-1(\dots)^{-2}$ soi (or quotient rule M1,numerator A1,denominator A1) attempt to verify or by integration as in (b)(ii)
(b)(i) $\frac{1}{P(2P-1)} = \frac{A}{P} + \frac{B}{2P-1}$ $= \frac{A(2P-1) + BP}{P(2P-1)}$ $\Rightarrow 1 = A(2P-1) + BP$ $P=0 \Rightarrow 1 = -A \Rightarrow A = -1$ $P = 1/2 \Rightarrow 1 = A \cdot 0 + 1/2 B \Rightarrow B = 2$ So $\frac{1}{P(2P-1)} = -\frac{1}{P} + \frac{2}{2P-1}$	M1 M1 A1 A1 [4]	correct partial fractions substituting values, equating coeffs or cover up rule $A = -1$ $B = 2$
(ii) $\frac{dP}{dt} = \frac{1}{2}(2P - P^2)\cos t$ $\Rightarrow \int \frac{1}{2P^2 - P} dP = \int \frac{1}{2} \cos t dt$ $\Rightarrow \int \left(\frac{2}{2P-1} - \frac{1}{P}\right) dP = \int \frac{1}{2} \cos t dt$ $\Rightarrow \ln(2P-1) - \ln P = 1/2 \sin t + c$ When $t = 0, P = 1$ $\Rightarrow \ln 1 - \ln 1 = 1/2 \sin 0 + c \Rightarrow c = 0$ $\Rightarrow \ln\left(\frac{2P-1}{P}\right) = \frac{1}{2} \sin t$ *	M1 A1 A1 B1 E1 [5]	separating variables $\ln(2P-1) - \ln P$ ft their A,B from (i) $1/2 \sin t$ finding constant = 0
(iii) $P_{\max} = \frac{1}{2-e^{1/2}} = 2.847$ $P_{\min} = \frac{1}{2-e^{-1/2}} = 0.718$	M1A1 M1A1 [4]	www www

8 (i) $\frac{dy}{dx} = \frac{10\cos\theta + 10\cos 2\theta}{-10\sin\theta - 10\sin 2\theta}$ $= -\frac{\cos\theta + \cos 2\theta}{\sin\theta + \sin 2\theta} *$ When $\theta = \pi/3$, $\frac{dy}{dx} = -\frac{\cos\pi/3 + \cos 2\pi/3}{\sin\pi/3 + \sin 2\pi/3}$ $= 0$ as $\cos\pi/3 = \frac{1}{2}$, $\cos 2\pi/3 = -\frac{1}{2}$ At A $x = 10\cos\pi/3 + 5\cos 2\pi/3$ $= 2\frac{1}{2}$ $y = 10\sin\pi/3 + 5\sin 2\pi/3 = 15\sqrt{3}/2$	M1 E1 B1 M1 A1 A1 [6]	$dy/d\theta \div dx/d\theta$ or solving $\cos\theta + \cos 2\theta = 0$ substituting $\pi/3$ into x or y $2\frac{1}{2}$ $15\sqrt{3}/2$ (condone 13 or better)
(ii) $x^2 + y^2 = (10\cos\theta + 5\cos 2\theta)^2 + (10\sin\theta + 5\sin 2\theta)^2$ $= 100\cos^2\theta + 100\cos\theta\cos 2\theta + 25\cos^2 2\theta$ $+ 100\sin^2\theta + 100\sin\theta\sin 2\theta + 25\sin^2 2\theta$ $= 100 + 100\cos(2\theta - \theta) + 25$ $= 125 + 100\cos\theta *$	B1 M1 DM1 E1 [4]	expanding $\cos 2\theta\cos\theta + \sin 2\theta\sin\theta = \cos(2\theta - \theta)$ or substituting for $\sin 2\theta$ and $\cos 2\theta$
(iii) Max $\sqrt{125+100} = 15$ min $\sqrt{125-100} = 5$	B1 B1 [2]	
(iv) $2\cos^2\theta + 2\cos\theta - 1 = 0$ $\cos\theta = \frac{-2 \pm \sqrt{12}}{4} = \frac{-2 \pm 2\sqrt{3}}{4}$ At B, $\cos\theta = \frac{-1 + \sqrt{3}}{2}$ $OB^2 = 125 + 50(-1 + \sqrt{3}) = 75 + 50\sqrt{3} = 161.6\dots$ $\Rightarrow OB = \sqrt{161.6\dots} = 12.7 \text{ (m)}$	M1 A1 M1 A1 [4]	quadratic formula or $\theta = 68.53^\circ$ or 1.20 radians, correct root selected or $OB = 10\sin\theta + 5\sin 2\theta$ ft their $\theta/\cos\theta$ oe cao

Paper B Comprehension

1)	M $(a\pi, 2a)$, $\theta=\pi$ N $(4a\pi, 0)$, $\theta=4\pi$	B1 B1	
2)	Compare the equations with equations given in text, $x = a\theta - b\sin\theta$, $y = b\cos\theta$	M1	Seeing $a=7$, $b=0.25$
	Wavelength = $2\pi a = 14\pi (\approx 44)$ Height = $2b = 0.5$	A1 B1	
3i)	Wavelength = $20 \Rightarrow a = \frac{10}{\pi} (\approx 3.18...)$ Height = $2 \Rightarrow b = 1$	B1 B1	
ii)	In this case, the ratio is observed to be 12:8 Trough length : Peak length = $\pi a + 2b : \pi a - 2b$ and this is $(10 + 2 \times 1) : (10 - 2 \times 1)$ So the curve is consistent with the parametric equations	B1 M1 A1	substituting
4i)	$x = a\theta$, $y = b\cos\theta$ is the sine curve V and $x = a\theta - b\sin\theta$, $y = b\cos\theta$ is the curtate cycloid U. The sine curve is above mid-height for half its wavelength (or equivalent)	B1	
ii)	$d = a\theta - (a\theta - b\sin\theta)$ $\theta = \pi/2$, $d = \left(\frac{\pi a}{2}\right) - \left(\frac{\pi a}{2} - b\right) = b$	M1 E1	Subtraction Using $\theta = \pi/2$
iii)	Because b is small compared to a , the two curves are close together.	M1 E1	Comparison attempted Conclusion
5)	Measurements on the diagram give Wavelength $\approx 3.5\text{cm}$, Height $\approx 0.8\text{cm}$ $\frac{\text{Wavelength}}{\text{Height}} \approx \frac{3.5}{0.8} = 4.375$ Since $4.375 < 7$, the wave will have become unstable and broken.	B1 M1 E1	measurements/reading ratio [18]