Mark Scheme 4726
June 2007

1 Correct formula with correct r — M1 — Allow $r^2 = 2\sin^2 3\theta$
Rewrite as $a + b\cos 6\theta$ — M1 — $a, b \neq 0$
Integrate their expression correctly — A1√ — From $a + b\cos 6\theta$
Get $\frac{1}{3}\pi$ — A1 — cao

2 (i) Expand to $\sin 2x \cos\frac{1}{4}\pi + \cos 2x \sin\frac{1}{4}\pi$ — B1
Clearly replace $\cos\frac{1}{4}\pi$, $\sin\frac{1}{4}\pi$ to A.G. — B1

(ii) Attempt to expand $\cos 2x$ — M1 — Allow $1 - 2x^2/2$
Attempt to expand $\sin 2x$ — M1 — Allow $2x - 2x^3/3$
Get $\frac{1}{2}\sqrt{2}\,(1 + 2x - 2x^2 - 4x^3/3)$ — A1 — Four correct unsimplified terms in any order; allow bracket; AEEF
SR Reasonable attempt at $f^{n}(0)$ for $n = 0$ to 3 M1
Attempt to replace their values in Maclaurin M1
Get correct answer only A1

3 (i) Express as $A/(x-1) + (Bx+C)/(x^2+9)$ — M1 — Allow $C=0$ here
Equate (x^2+9x) to $A(x^2+9) + (Bx+C)(x-1)$ — M1√ — May imply above line; on their P.F.
Sub. for x or equate coeff. — M1 — Must lead to at least 3 coeff.; allow cover-up method for A
Get A=1, B=0, C=9 — A1 — cao from correct method

(ii) Get $A\ln(x-1)$ — B1√ — On their A
Get $C/3\tan^{-1}(x/3)$ — B1√ — On their C; condone no constant; ignore any $B \neq 0$

4 (i) Reasonable attempt at product rule — M1 — Two terms seen
Derive or quote diff. of $\cos^{-1}x$ — M1 — Allow +
Get $-x^2(1 - x^2)^{-\frac{1}{2}} + (1 - x^2)^{\frac{1}{2}} + (1 - x^2)^{-\frac{1}{2}}$ — A1
Tidy to $2(1 - x^2)^{\frac{1}{2}}$ — A1 — cao

(ii) Write down integral from (i) — B1 — On any $k\sqrt{(1-x^2)}$
Use limits correctly — M1 — In any reasonable integral
Tidy to $\frac{1}{2}\pi$ — A1
SR Reasonable sub. B1
Replace for new variable and attempt to integrate (ignore limits) M1
Clearly get $\frac{1}{2}\pi$ A1

5	(i)	Attempt at parts on $\int 1\,(\ln x)^n\,dx$	M1	Two terms seen
		Get $x\,(\ln x)^n - \int n\,(\ln x)^{n-1}\,dx$	A1	
		Put in limits correctly in line above	M1	
		Clearly get A.G.	A1	ln e =1, ln1 =0 seen or implied
	(ii)	Attempt I_3 to I_2 as $I_3 = e - 3I_2$	M1	
		Continue sequence in terms of In	A1	$I_2 = e-2I_1$ and/or $I_1 = e-I_0$
		Attempt I_0 or I_1	M1	(I_0 =e-1, I_1=1)
		Get 6 – 2e	A1	cao
6	(i)	Area under graph (= $\int 1/x^2\,dx$, 1 to n+1) < Sum of rectangles (from 1 to n)	B1	Sum (total) seen or implied eg diagram; accept areas (of rectangles)
		Area of each rectangle = Width x Height = 1 x $1/x^2$	B1	Some evidence of area worked out – seen or implied
	(ii)	Indication of new set of rectangles	B1	
		Similarly, area under graph from 1 to n > sum of areas of rectangles from 2 to n	B1	Sum (total) seen or implied
		Clear explanation of A.G.	B1	Diagram; use of left-shift of previous areas
	(iii)	Show complete integrations of RHS, using correct, different limits	M1	Reasonable attempt at $\int x^{-2}\,dx$
		Correct answer, using limits, to one integral	A1	
		Add 1 to their second integral to get complete series	M1	
		Clearly arrive at A.G.	A1	
	(iv)	Get one limit	B1	Quotable
		Get both 1 and 2	B1	Quotable; limits only required

7 (i) Use correct definition of cosh or sinh x — B1 — Seen anywhere in (i)
Attempt to mult. their cosh/sinh — M1
Correctly mult. out and tidy — A1√
Clearly arrive at A.G. — A1 — Accept e^{x-y} and e^{y-x}

(ii) Get $\cosh(x-y) = 1$ — M1
Get or imply $(x-y) = 0$ to A.G. — A1

(iii) Use $\cosh^2 x = 9$ or $\sinh^2 x = 8$ — B1
Attempt to solve cosh x = 3 (not –3) or sinh $x = \pm\sqrt{8}$ (allow $+\sqrt{8}$ or $-\sqrt{8}$ only) — M1 — $x = \ln(3+\sqrt{8})$ from formulae book or from basic cosh definition
Get at least one x solution correct — A1
Get both solutions correct, x and y — A1 — x, y = $\ln(3 \pm 2\sqrt{2})$; AEEF

SR Attempt tanh = sinh/cosh — B1
Get tanh $x = \pm\sqrt{8}/3$ (+ or -) — M1
Get at least one sol. correct — A1
Get both solutions correct — A1

SR Use exponential definition — B1
Get quadratic in e^x or e^{2x} — M1
Solve for one correct x — A1
Get both solutions, x and y — A1

8 (i) $x_2 = 0.1890$ — B1
$x_3 = 0.2087$ — B1√ — From their x_1 (or any other correct)
$x_4 = 0.2050$ — B1√ — Get at least two others correct, all to a minimum of 4 d.p.
$x_5 = 0.2057$
$x_6 = 0.2055$
$x_7 (= x_8) = 0.2056$ (to x_7 minimum)
$\alpha = 0.2056$ — B1 — cao; answer may be retrieved despite some errors

(ii) Attempt to diff. f(x) — M1 — $k/(2+x)^3$
Use α to show $f'(\alpha) \neq 0$ — A1√ — Clearly seen, or explain $k/(2+x)^3 \neq 0$ as $k \neq 0$; allow ± 0.1864

SR Translate $y=1/x^2$ — M1
State/show $y=1/x^2$ has no TP — A1

(iii) $\delta_3 = -0.0037$ (allow –0.004) — B1√ — Allow ±, from their x_4 and x_3

(iv) Develop from $\delta_{10} = f'(\alpha)\,\delta_9$ etc. to get δ_i or quote $\delta_{10} = \delta_3 f'(\alpha)^7$ — M1 — Or any δ_i eg use $\delta_9 = x_{10} - x_9$
Use their δ_i and $f'(\alpha)$ — M1
Get 0.000000028 — A1 — Or answer that rounds to ± 0.00000003

9 (i) Quote $x = a$ B1

Attempt to divide out M1 Allow M1 for y=x here; allow

A1 $(x\text{-}a) + k/(x\text{-}a)$ seen or implied

Get $y = x - a$ A1 Must be equations

(ii) Attempt at quad. in x (=0) M1

Use $b^2 - 4ac \geq 0$ for real x M1 Allow >

Get $y^2 + 4a^2 \geq 0$ A1

State/show their quad. is always >0 B1 Allow ≥

(iii) B1√ Two asymptotes from (i) (need not be labelled)

B1 Both crossing points

B1√ Approaches – correct shape

SR Attempt diff. by quotient/product rule M1

Get quadratic in x for $dy/dx = 0$ and note $b^2 - 4ac < 0$ A1

Consider horizontal asymptotes B1

Fully justify answer B1