Mark Scheme 4734
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1	$\int_0^1 a\,dx \quad + \quad \int_1^\infty \frac{a}{x^2}\,dx = 1$	M1		For sum of integrals =1
	$[ax]_0^1 \quad + \quad \left[-\frac{a}{x^3}\right]_1^\infty = 1$	A1		For second integral.
	$a \quad + \quad a = 1$	A1		For second a
	$a = ½$	A1	**4**	Or from F(x) M1A1 then F(∞)=1 M1, a=1/2 A1
2	(i) $\overline{X}_I \sim N(5, \frac{0.7^2}{20})$	B1		If no parameters allow in (ii)
	$\overline{X}_E \sim N(4.5, \frac{0.5^2}{25})$	B1	**2**	If 0.7/20, 0.5/25 then B1 for both, with means in (ii)
	(ii) Use $\overline{X}_I - \overline{X}_E \sim N(0.5, \sigma^2)$	M1A1		OR $\overline{X}_I - \overline{X}_E - 1 \sim N(-0.5, \sigma^2)$
	$\sigma^2 = 0.49/20 + 0.25/25$	B1		cao
	1- $\Phi([1-0.5]/\sigma)$	M1		RH probability implied. If 0.7, 0.5
	= 0.0036 or 0.0035	A1	**5**	in σ^2, M1A1B0M1A1 for 0.165
3	Assumes differences form a random sample from a normal distribution.		B1	
	H_0: $\mu = 0$, H_1: $\mu > 0$	B1		Other letters if defined; or in words
	$\overline{x} = 17.2/12$; $s^2 = 10.155$ AEF	B1B1		Or $(12/11)(136.36/12-(17.2/12)^2)$aef
	EITHER: $t = \frac{\overline{x}}{\sqrt{s^2/12}}$ (+ or -)	M1		With 12 or 9.309/11
	=1.558	A1		Must be positive. Accept 1.56
	1.363 seen	B1		
	1.558 > 1.363, so reject H_0 and accept that there that the readings from the aneroid device overestimate blood pressure on average	B1√		Allow CV of 1.372 or 1.356 evidence Explicit comparison of CV(not - with +) and conclusion in context.
	OR: For critical region or critical value of $\overline{x}$			
	$1.363\sqrt{(s^2/12)}$	M1B1		B1 for correct ***t***
	Giving 1.25(3)	A1		
	Compare 1.43(3) with 1.25(3)			
	Conclusion in context	B1√	**8**	

4	(i) Proper P F Trial P **31** **11** 42 Trial F **5** 13 **18** 36 **24** 60	B1 B1	**2**	Two correct Others correct
	(ii) (H_0: Trial results and Proper results are independent.)			
	E-values: 25.2 16.8	M1		One correct. Ft marginals in (i)
	10.8 7.2	A1		All correct
	$\chi^2 = 5.3^2(25.2^{-1}+10.8^{-1}+16.8^{-1}+7.2^{-1})$	M1		Allow two errors
		A1		With Yates' correction
	$= 9.289$	A1		art 9.29
	Compare correctly with 7.8794	M1		Or 7.88
	There is evidence that results are not independent.	A1 √	**7**	Ft χ^2_{calc}.
5	(i) $e^{-\mu} = 0.45$	M1		
	$\mu_G = 0.799 \approx 0.80$ AG	A1	**2**	0.799 or 0.798 or better seen
	(ii) $\mu_U \approx 1.8$	B1		
	Total, $T \sim \text{Po}(2.6)$	M1		May be implied by answer 0.264
	P(>3) = 0.264	A1	**3**	From table or otherwise
	(iii) $e^{-2.6}2.6^6/6!$	B1		Or 0.318 from table
	$e^{-5.2}5.2^4/4!$	B1		
	Multiply two probabilities	M1		
	Answers rounding to 0.0053 or 0.0054	A1	4	

6	(i) $\hat{p} = 62/200 = 0.31$	B1		aef
	Use $\hat{p}_\alpha \pm z\sqrt{\frac{\hat{p}_\alpha(1-\hat{p}_\alpha)}{200}}$	M1		With 200 or 199
	z=1.96	B1		Seen
	Correct variance estimate	A1√		ft $\hat{p}$
	(0.2459,0.3741)	A1	**5**	art (0.246,0.374)
	(ii)EITHER: Sample proportion has an approximate normal distribution OR: Variance is an estimate	B1	**1**	Not $\hat{p}$ is an estimate, unless variance mentioned
	(iii) H_0: $p_\alpha = p_\beta$, H_1: $p_\alpha \neq p_\beta$ $\hat{p}$ =(62+35)/(200+150)	B1		aef
	EITHER: $z = (\pm)\frac{62/200 - 35/150}{\sqrt{\hat{p}\hat{q}(200^{-1}+150^{-1})}}$	M1		s^2 with, $\hat{p}, 200, 150$ (or 199,149)
		B1√		Evidence of correct variance estimate. Ft $\hat{p}$
	=1.586	A1		Rounding to 1.58 or 1.59
	(-1.96 <) 1.586 < 1.96	M1		Correct comparison with ± 1.96
	Do not reject H_0- there is insufficient evidence of a difference in proportions.	A1		SR: If variance $p_1q_1/n_1+p_2q_2/n_2$ used then: B0M1B0A1(for z=1.61 or 1.62)M1A1 Max 4/6.
	OR: $p_{s\alpha} - p_{s\beta} = zs$	M1		
	s = $\sqrt{(0.277\times0.723(200^{-1}+150^{-1}))}$	B1√		Ft $\hat{p}$
	CV of $p_{s\alpha} - p_{s\beta}$ = 0.0948 or 0.095	A1		
	Compare $p_{s\alpha} - p_{s\beta}$ = 0.0767 with their 0.0948	M1		
	Do not reject H_0 and accept that there is insufficient evidence of a difference in proportions	A1		Conditional on z=1.96
			6	

7	(i) $G(y) = P(Y \le y)$	M1		May be implied by following line
	$= P(X^2 \ge 1/y)$ [or $P(X > 1/\sqrt{y})$]	A1		Accept strict inequalities
	$= 1 - F(1/\sqrt{y})$	A1		
	$= \begin{cases} 0 & y \le 0, \\ y^2 & 0 \le y \le 1, \\ (1 & y > 1.) \end{cases}$	A1	**4**	Or $F(x)=P(X \le x) = P(Y \ge 1/x^2)$ M1 $=1 - P(Y<1/x^2)$ A1 $=1-G(y)$;etc A1 A1
	(ii) Differentiate their $G(y)$	M1		
	to obtain $g(y)= 2y$ for $0 < y \le 1$ AG	A1	**2**	Only from G correctly obtained
	(iii) $\int_0^1 2y(\sqrt[3]{y})dy$	M1		Unsimplified, but with limits
	$=[6y^{7/3}/7]$	B1		OR: Find f(x), $\int_1^\infty x^{-2/3} f(x)dx$ M1
	$= \ ^6/_7$	A1	**3**	$=[4x^{-14/3}/(14/3)]$; $^6/_7$ B1A1 OR: Find H(z), $Z= Y^{1/3}$
8	(i) $P(20 \le y < 25) = \Phi(0) - \Phi(-5/\sqrt{(20)})$	M1		
	Multiply by 50	A1		
	to give 18.41 AG	A1		
	18.41 for $25 \le y < 30$ and 6.59 for $y < 20$, $y \ge 30$	A1	**4**	
	(ii) H_0: N(25,20) fits data	B1		OR $Y \sim N(25,20)$
	$\chi^2 = 3.59^2/6.59 + 8.59^2/18.41 + 6.41^2/18.41 + 1.41^2/6.59$	M1√		ft values from (i)
	$=8.497$	A1		art 8.5
	$8.497 > 7.815$	M1		
	Accept that N(25,20) is not a good fit	A1	**5**	
	(iii) Use $24.91 \pm z\sqrt{(20/50)}$	M1		With $\sqrt{(20/50)}$
	z = 2.326	B1		
	(23.44,26.38)	A1	**3**	art (23.4,26.4) Must be interval
	(iv) No- Sample size large enough to apply CLT	B1		Refer to large sample size
	Sample mean will be (approximately) normally distributed whatever the distribution of Y	B1	**2**	Refer to normality of sample mean