

ADVANCED GCE UNIT

4726/01

Further Pure Mathematics 2
THURSDAY 7 JUNE 2007

Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages) List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer all the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are reminded of the need for clear presentation in your answers.

1 The equation of a curve, in polar coordinates, is

$$r = 2\sin 3\theta$$
, for $0 \le \theta \le \frac{1}{3}\pi$.

Find the exact area of the region enclosed by the curve between $\theta = 0$ and $\theta = \frac{1}{3}\pi$. [4]

- 2 (i) Given that $f(x) = \sin(2x + \frac{1}{4}\pi)$, show that $f(x) = \frac{1}{2}\sqrt{2}(\sin 2x + \cos 2x)$. [2]
 - (ii) Hence find the first four terms of the Maclaurin series for f(x). [You may use appropriate results given in the List of Formulae.] [3]
- 3 It is given that $f(x) = \frac{x^2 + 9x}{(x-1)(x^2+9)}$.
 - (i) Express f(x) in partial fractions. [4]

(ii) Hence find
$$\int f(x) dx$$
. [2]

4 (i) Given that

$$y = x\sqrt{1 - x^2} - \cos^{-1} x,$$

find
$$\frac{dy}{dx}$$
 in a simplified form. [4]

- (ii) Hence, or otherwise, find the exact value of $\int_0^1 2\sqrt{1-x^2} \, dx$. [3]
- 5 It is given that, for non-negative integers n,

$$I_n = \int_1^e (\ln x)^n \, \mathrm{d}x.$$

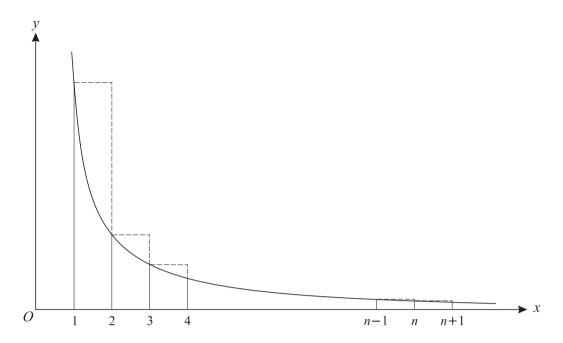
(i) Show that, for $n \ge 1$,

$$I_n = e - nI_{n-1}.$$
 [4]

(ii) Find I_3 in terms of e. [4]

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The diagram shows the curve with equation $y = \frac{1}{x^2}$ for x > 0, together with a set of *n* rectangles of unit width, starting at x = 1.

(i) By considering the areas of these rectangles, explain why

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{n^2} > \int_1^{n+1} \frac{1}{x^2} dx.$$
 [2]

(ii) By considering the areas of another set of rectangles, explain why

$$\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots + \frac{1}{n^2} < \int_1^n \frac{1}{x^2} \, \mathrm{d}x.$$
 [3]

(iii) Hence show that

$$1 - \frac{1}{n+1} < \sum_{r=1}^{n} \frac{1}{r^2} < 2 - \frac{1}{n}.$$
 [4]

(iv) Hence give bounds between which
$$\sum_{r=1}^{\infty} \frac{1}{r^2}$$
 lies. [2]

7 (i) Using the definitions of hyperbolic functions in terms of exponentials, prove that

$$\cosh x \cosh y - \sinh x \sinh y = \cosh(x - y).$$
 [4]

(ii) Given that $\cosh x \cosh y = 9$ and $\sinh x \sinh y = 8$, show that x = y. [2]

(iii) Hence find the values of x and y which satisfy the equations given in part (ii), giving the answers in logarithmic form. [4]

- 8 The iteration $x_{n+1} = \frac{1}{(x_n + 2)^2}$, with $x_1 = 0.3$, is to be used to find the real root, α , of the equation $x(x+2)^2 = 1$.
 - (i) Find the value of α , correct to 4 decimal places. You should show the result of each step of the iteration process. [4]

(ii) Given that
$$f(x) = \frac{1}{(x+2)^2}$$
, show that $f'(\alpha) \neq 0$. [2]

- (iii) The difference, δ_r , between successive approximations is given by $\delta_r = x_{r+1} x_r$. Find δ_3 . [1]
- (iv) Given that $\delta_{r+1} \approx f'(\alpha)\delta_r$, find an estimate for δ_{10} . [3]
- 9 It is given that the equation of a curve is

$$y = \frac{x^2 - 2ax}{x - a},$$

where a is a positive constant.

- (i) Find the equations of the asymptotes of the curve. [4]
- (ii) Show that y takes all real values. [4]
- (iii) Sketch the curve $y = \frac{x^2 2ax}{x a}$. [3]

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