INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

1   The equation of a curve, in polar coordinates, is

$$r = 2\sin 3\theta, \quad \text{for } 0 \leqslant \theta \leqslant \tfrac{1}{3}\pi.$$

Find the exact area of the region enclosed by the curve between $\theta = 0$ and $\theta = \tfrac{1}{3}\pi$.   [4]

2   **(i)** Given that $f(x) = \sin(2x + \tfrac{1}{4}\pi)$, show that $f(x) = \tfrac{1}{2}\sqrt{2}(\sin 2x + \cos 2x)$.   [2]

   (ii) Hence find the first four terms of the Maclaurin series for $f(x)$. [You may use appropriate results given in the List of Formulae.]   [3]

3   It is given that $f(x) = \dfrac{x^2 + 9x}{(x - 1)(x^2 + 9)}$.

   (i) Express $f(x)$ in partial fractions.   [4]

   (ii) Hence find $\displaystyle\int f(x)\, dx$.   [2]

4   **(i)** Given that

$$y = x\sqrt{1 - x^2} - \cos^{-1} x,$$

find $\dfrac{dy}{dx}$ in a simplified form.   [4]

   (ii) Hence, or otherwise, find the exact value of $\displaystyle\int_0^1 2\sqrt{1 - x^2}\, dx$.   [3]

5   It is given that, for non-negative integers n,
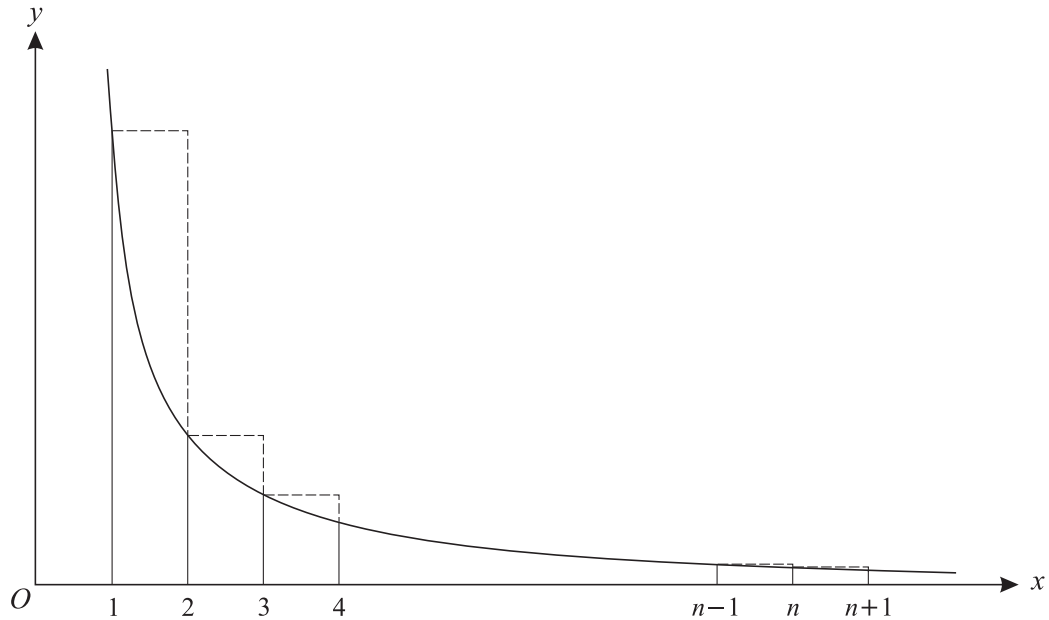
$$I_n = \int_1^e (\ln x)^n\, dx.$$

   (i) Show that, for $n \geqslant 1$,

$$I_n = e - nI_{n-1}.$$ [4]

   (ii) Find I_3 in terms of e.   [4]

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The diagram shows the curve with equation $y = \dfrac{1}{x^2}$ for $x > 0$, together with a set of n rectangles of unit width, starting at $x = 1$.

  (i) By considering the areas of these rectangles, explain why

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{n^2} > \int_1^{n+1} \frac{1}{x^2}\,dx.$$

               [2]

  (ii) By considering the areas of another set of rectangles, explain why

$$\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots + \frac{1}{n^2} < \int_1^{n} \frac{1}{x^2}\,dx.$$

               [3]

  (iii) Hence show that

$$1 - \frac{1}{n+1} < \sum_{r=1}^{n} \frac{1}{r^2} < 2 - \frac{1}{n}.$$

               [4]

  (iv) Hence give bounds between which $\displaystyle\sum_{r=1}^{\infty} \frac{1}{r^2}$ lies.

               [2]

7   **(i)** Using the definitions of hyperbolic functions in terms of exponentials, prove that

$$\cosh x \cosh y - \sinh x \sinh y = \cosh(x - y).$$

               [4]

  (ii) Given that $\cosh x \cosh y = 9$ and $\sinh x \sinh y = 8$, show that $x = y$.

               [2]

  (iii) Hence find the values of x and y which satisfy the equations given in part **(ii)**, giving the answers in logarithmic form.

               [4]

8 The iteration $x_{n+1} = \dfrac{1}{(x_n + 2)^2}$, with $x_1 = 0.3$, is to be used to find the real root, α, of the equation $x(x + 2)^2 = 1$.

   (i) Find the value of α, correct to 4 decimal places. You should show the result of each step of the iteration process. [4]

   (ii) Given that $f(x) = \dfrac{1}{(x + 2)^2}$, show that $f'(\alpha) \neq 0$. [2]

   (iii) The difference, δ_r, between successive approximations is given by $\delta_r = x_{r+1} - x_r$. Find δ_3. [1]

   (iv) Given that $\delta_{r+1} \approx f'(\alpha)\delta_r$, find an estimate for δ_{10}. [3]

9 It is given that the equation of a curve is

$$y = \frac{x^2 - 2ax}{x - a},$$

where a is a positive constant.

   (i) Find the equations of the asymptotes of the curve. [4]

   (ii) Show that y takes all real values. [4]

   (iii) Sketch the curve $y = \dfrac{x^2 - 2ax}{x - a}$. [3]