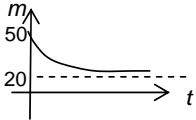


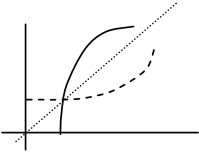
4753 (C3) Methods for Advanced Mathematics

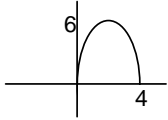
Section A

<p>1 $2x-1 \leq 3$ $\Rightarrow -3 \leq 2x-1 \leq 3$ $\Rightarrow -2 \leq 2x \leq 4$ $\Rightarrow -1 \leq x \leq 2$ or $(2x-1)^2 \leq 9$ $\Rightarrow 4x^2 - 4x - 8 \leq 0$ $\Rightarrow (4)(x+1)(x-2) \leq 0$ $\Rightarrow -1 \leq x \leq 2$</p>	<p>M1 A1 M1 A1 M1 A1 A1 A1 [4]</p>	<p>$2x-1 \leq 3$ (or =) $x \leq 2$ $2x-1 \geq -3$ (or =) $x \geq -1$ squaring and forming quadratic = 0 (or \leq) factorising or solving to get $x = -1, 2$ $x \geq -1$ $x \leq 2$ (www)</p>
<p>2 Let $u = x$, $dv/dx = e^{3x} \Rightarrow v = e^{3x}/3$ $\Rightarrow \int x e^{3x} dx = \frac{1}{3} x e^{3x} - \int \frac{1}{3} e^{3x} \cdot 1 dx$ $= \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + c$</p>	<p>M1 A1 A1 B1 [4]</p>	<p>parts with $u = x$, $dv/dx = e^{3x} \Rightarrow v$ $= \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x}$ $+c$</p>
<p>3 (i) $f(-x) = f(x)$ Symmetrical about Oy.</p>	<p>B1 B1 [2]</p>	
<p>(ii) (A) even (B) neither (C) odd</p>	<p>B1 B1 B1 [3]</p>	
<p>4 Let $u = x^2 + 2 \Rightarrow du = 2x dx$ $\int_1^4 \frac{x}{x^2+2} dx = \int_3^{18} \frac{1/2}{u} du$ $= \frac{1}{2} [\ln u]_3^{18}$ $= \frac{1}{2} (\ln 18 - \ln 3)$ $= \frac{1}{2} \ln(18/3)$ $= \frac{1}{2} \ln 6^*$</p>	<p>M1 A1 M1 E1 [4]</p>	<p>$\int \frac{1/2}{u} du$ or $k \ln(x^2 + 1)$ $\frac{1}{2} \ln u$ or $\frac{1}{2} \ln(x^2 + 2)$ substituting correct limits (u or x) must show working for $\ln 6$</p>
<p>5 $y = x^2 \ln x$ $\Rightarrow \frac{dy}{dx} = x^2 \cdot \frac{1}{x} + 2x \ln x$ $= x + 2x \ln x$ $dy/dx = 0$ when $x + 2x \ln x = 0$ $\Rightarrow x(1 + 2 \ln x) = 0$ $\Rightarrow \ln x = -\frac{1}{2}$ $\Rightarrow x = e^{-1/2} = 1/\sqrt{e}^*$</p>	<p>M1 B1 A1 M1 M1 E1 [6]</p>	<p>product rule $d/dx (\ln x) = 1/x$ soi oe their deriv = 0 or attempt to verify $\ln x = -\frac{1}{2} \Rightarrow x = e^{-1/2}$ or $\ln(1/\sqrt{e}) = -\frac{1}{2}$</p>

6(i) Initial mass = $20 + 30 e^0 = 50$ grams Long term mass = 20 grams	M1A1 B1 [3]	
(ii) $30 = 20 + 30 e^{-0.1t}$ $\Rightarrow e^{-0.1t} = 1/3$ $\Rightarrow -0.1t = \ln(1/3) = -1.0986\dots$ $\Rightarrow t = 11.0$ mins	M1 M1 A1 [3]	anti-logging correctly 11, 11.0, 10.99, 10.986 (not more than 3 d.p)
(iii) 	B1 B1 [2]	correct shape through (0, 50) – ignore negative values of t $\rightarrow 20$ as $t \rightarrow \infty$
7 $x^2 + xy + y^2 = 12$ $\Rightarrow 2x + x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = 0$ $\Rightarrow (x + 2y) \frac{dy}{dx} = -2x - y$ $\Rightarrow \frac{dy}{dx} = -\frac{2x + y}{x + 2y}$	M1 B1 A1 M1 A1 [5]	Implicit differentiation $x \frac{dy}{dx} + y$ correct equation collecting terms in dy/dx and factorising oe cao

Section B

8(i) $y = 1/(1+\cos\pi/3) = 2/3.$	B1 [1]	or 0.67 or better
(ii) $f'(x) = -1(1+\cos x)^{-2} \cdot -\sin x$ $= \frac{\sin x}{(1+\cos x)^2}$ When $x = \pi/3$, $f'(x) = \frac{\sin(\pi/3)}{(1+\cos(\pi/3))^2}$ $= \frac{\sqrt{3}/2}{(1\frac{1}{2})^2} = \frac{\sqrt{3}}{2} \times \frac{4}{9} = \frac{2\sqrt{3}}{9}$	M1 B1 A1 M1 A1 [5]	chain rule or quotient rule $d/dx (\cos x) = -\sin x$ soi correct expression substituting $x = \pi/3$ oe or 0.38 or better. (0.385, 0.3849)
(iii) deriv = $\frac{(1+\cos x)\cos x - \sin x \cdot (-\sin x)}{(1+\cos x)^2}$ $= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$ $= \frac{\cos x + 1}{(1+\cos x)^2}$ $= \frac{1}{1+\cos x} *$ Area = $\int_0^{\pi/3} \frac{1}{1+\cos x} dx$ $= \left[\frac{\sin x}{1+\cos x} \right]_0^{\pi/3}$ $= \frac{\sin \pi/3}{1+\cos \pi/3} - (-0)$ $= \frac{\sqrt{3}}{2} \times \frac{2}{3} = \frac{\sqrt{3}}{3}$	M1 A1 M1dep E1 B1 M1 A1 cao [7]	Quotient or product rule – condone $uv' - u'v$ for M1 correct expression $\cos^2 x + \sin^2 x = 1$ used dep M1 www substituting limits or $1/\sqrt{3}$ - must be exact
(iv) $y = 1/(1+\cos x) \quad x \leftrightarrow y$ $x = 1/(1+\cos y)$ $\Rightarrow 1+\cos y = 1/x$ $\Rightarrow \cos y = 1/x - 1$ $\Rightarrow y = \arccos(1/x - 1) *$ Domain is $\frac{1}{2} \leq x \leq 1$ 	M1 A1 E1 B1 B1 [5]	attempt to invert equation www reasonable reflection in $y = x$

<p>9 (i) $y = \sqrt{4 - x^2}$ $\Rightarrow y^2 = 4 - x^2$ $\Rightarrow x^2 + y^2 = 4$ which is equation of a circle centre O radius 2 Square root does not give negative values, so this is only a semi-circle.</p>	M1 A1 B1 [3]	squaring $x^2 + y^2 = 4$ + comment (correct) oe, e.g. f is a function and therefore single valued
<p>(ii) (A) Grad of OP = b/a \Rightarrow grad of tangent = $-\frac{a}{b}$</p> <p>(B) $f'(x) = \frac{1}{2}(4 - x^2)^{-1/2} \cdot (-2x)$ $= -\frac{x}{\sqrt{4 - x^2}}$ $\Rightarrow f'(a) = -\frac{a}{\sqrt{4 - a^2}}$</p> <p>(C) $b = \sqrt{4 - a^2}$ so $f'(a) = -\frac{a}{b}$ as before</p>	M1 A1 M1 A1 B1 E1 [6]	 chain rule or implicit differentiation oe substituting a into their $f'(x)$
<p>(iii) Translation through $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ followed by</p> <p>stretch scale factor 3 in y-direction</p> 	M1 A1 M1 A1 M1 A1 [6]	Translation in x-direction through $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ or 2 to right ('shift', 'move' M1 A0) $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ alone is SC1 stretch in y-direction (condone y 'axis') (scale) factor 3 elliptical (or circular) shape through (0, 0) and (4, 0) and (2, 6) (soi) -1 if whole ellipse shown
<p>(iv) $y = 3f(x - 2)$ $= 3\sqrt{4 - (x - 2)^2}$ $= 3\sqrt{4 - x^2 + 4x - 4}$ $= 3\sqrt{4x - x^2}$ $\Rightarrow y^2 = 9(4x - x^2)$ $\Rightarrow 9x^2 + y^2 = 36x$ *</p>	M1 A1 E1 [3]	or substituting $3\sqrt{4 - (x - 2)^2}$ oe for y in $9x^2 + y^2$ $4x - x^2$ www