

4754 (C4) Applications of Advanced Mathematics

Section A

1 $\frac{x}{x^2-4} + \frac{2}{x+2} = \frac{x}{(x-2)(x+2)} + \frac{2}{x+2}$ $= \frac{x+2(x-2)}{(x+2)(x-2)}$ $= \frac{3x-4}{(x+2)(x-2)}$	M1 M1 A1 [3]	combining fractions correctly factorising and cancelling (may be $3x^2+2x-8$)
2 $V = \int_0^1 \pi y^2 dx = \int_0^1 \pi(1+e^{2x}) dx$ $= \pi \left[x + \frac{1}{2} e^{2x} \right]_0^1$ $= \pi \left(1 + \frac{1}{2} e^2 - \frac{1}{2} \right)$ $= \frac{1}{2} \pi (1+e^2) *$	M1 B1 M1 E1 [4]	must be π x their y^2 in terms of x $\left[x + \frac{1}{2} e^{2x} \right]$ only substituting both x limits in a function of x www
3 $\cos 2\theta = \sin \theta$ $\Rightarrow 1 - 2\sin^2 \theta = \sin \theta$ $\Rightarrow 1 - \sin \theta - 2\sin^2 \theta = 0$ $\Rightarrow (1 - 2\sin \theta)(1 + \sin \theta) = 0$ $\Rightarrow \sin \theta = \frac{1}{2} \text{ or } -1$ $\Rightarrow \theta = \pi/6, 5\pi/6, 3\pi/2$	M1 M1 A1 M1 A1 A2,1,0 [7]	$\cos 2\theta = 1 - 2\sin^2 \theta$ oe substituted forming quadratic(in one variable) =0 correct quadratic www factorising or solving quadratic $\frac{1}{2}, -1$ oe www cao penalise extra solutions in the range
4 $\sec \theta = x/2, \tan \theta = y/3$ $\sec^2 \theta = 1 + \tan^2 \theta$ $\Rightarrow x^2/4 = 1 + y^2/9$ $\Rightarrow x^2/4 - y^2/9 = 1 *$ OR $x^2/4 - y^2/9 = 4\sec^2 \theta/4 - 9\tan^2 \theta/9$ $= \sec^2 \theta - \tan^2 \theta = 1$	M1 M1 E1 [3]	$\sec^2 \theta = 1 + \tan^2 \theta$ used (oe, e.g. converting to sines and cosines and using $\cos^2 \theta + \sin^2 \theta = 1$) eliminating θ (or x and y) www
5(i) $dx/du = 2u, dy/du = 6u^2$ $\Rightarrow \frac{dy}{dx} = \frac{dy/du}{dx/du} = \frac{6u^2}{2u}$ $= 3u$ OR $y = 2(x-1)^{3/2}, dy/dx = 3(x-1)^{1/2} = 3u$	B1 M1 A1 [3]	both $2u$ and $6u^2$ B1($y=f(x)$), M1 differentiation, A1
(ii) At $(5, 16), u = 2$ $\Rightarrow dy/dx = 6$	M1 A1 [2]	cao

6(i) $(1+4x^2)^{-\frac{1}{2}} = 1 - \frac{1}{2} \cdot 4x^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (4x^2)^2 + \dots$ $= 1 - 2x^2 + 6x^4 + \dots$ Valid for $-1 < 4x^2 < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$	M1 A1 A1 M1A1 [5]	binomial expansion with $p = -1/2$ $1 - 2x^2 \dots$ $+ 6x^4$
(ii) $\frac{1-x^2}{\sqrt{1+4x^2}} = (1-x^2)(1-2x^2+6x^4+\dots)$ $= 1 - 2x^2 + 6x^4 - x^2 + 2x^4 + \dots$ $= 1 - 3x^2 + 8x^4 + \dots$	M1 A1 A1 [3]	substituting their $1 - 2x^2 + 6x^4 + \dots$ and expanding ft their expansion (of three terms) cao
7 $\sqrt{3} \sin x - \cos x = R \sin(x - \alpha)$ $= R(\sin x \cos \alpha - \cos x \sin \alpha)$ $\Rightarrow \sqrt{3} = R \cos \alpha, 1 = R \sin \alpha$ $\Rightarrow R^2 = 3 + 1 = 4 \Rightarrow R = 2$ $\tan \alpha = 1/\sqrt{3}$ $\Rightarrow \alpha = \pi/6$ $\Rightarrow y = 2 \sin(x - \pi/6)$ Max when $x - \pi/6 = \pi/2 \Rightarrow x = \pi/6 + \pi/2 = 2\pi/3$ max value $y = 2$ So maximum is $(2\pi/3, 2)$	M1 B1 M1 A1 B1 B1 [6]	correct pairs soi $R = 2$ ft cao www cao ft their R SC B1 $(2, 2\pi/3)$ no working

Section B

8(i) At A: $3 \times 0 + 2 \times 0 + 20 \times (-15) + 300 = 0$ At B: $3 \times 100 + 2 \times 0 + 20 \times (-30) + 300 = 0$ At C: $3 \times 0 + 2 \times 100 + 20 \times (-25) + 300 = 0$ So ABC has equation $3x + 2y + 20z + 300 = 0$	M1 A2,1,0 [3]	substituting co-ords into equation of plane... for ABC OR using two vectors in the plane form vector product M1A1 then $3x + 2y + 20z = c = -300$ A1 OR using vector equation of plane M1,elim both parameters M1, A1
(ii) $\overrightarrow{DE} = \begin{pmatrix} 100 \\ 0 \\ -10 \end{pmatrix}$ $\overrightarrow{DF} = \begin{pmatrix} 0 \\ 100 \\ 5 \end{pmatrix}$ $\begin{pmatrix} 100 \\ 0 \\ -10 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 20 \end{pmatrix} = 100 \times 2 + 0 \times -1 + -10 \times 20 = 200 - 200 = 0$ $\begin{pmatrix} 0 \\ 100 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 20 \end{pmatrix} = 0 \times 2 + 100 \times -1 + 5 \times 20 = -100 + 100 = 0$ Equation of plane is $2x - y + 20z = c$ At D (say) $c = 20 \times -40 = -800$ So equation is $2x - y + 20z + 800 = 0$	B1B1 B1 B1 M1 A1 [6]	need evaluation need evaluation
(iii) Angle is θ, where $\cos \theta = \frac{\begin{pmatrix} 2 \\ -1 \\ 20 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ 20 \end{pmatrix}}{\sqrt{2^2 + (-1)^2 + 20^2} \sqrt{3^2 + 2^2 + 20^2}} = \frac{404}{\sqrt{405} \sqrt{413}}$ $\Rightarrow \theta = 8.95^\circ$	M1 A1 A1 A1cao [4]	formula with correct vectors top bottom (or 0.156 radians)
(iv) RS: $\mathbf{r} = \begin{pmatrix} 15 \\ 34 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ 20 \end{pmatrix}$ $= \begin{pmatrix} 15 + 3\lambda \\ 34 + 2\lambda \\ 20\lambda \end{pmatrix}$ $\Rightarrow 3(15 + 3\lambda) + 2(34 + 2\lambda) + 20.20\lambda + 300 = 0$ $\Rightarrow 45 + 9\lambda + 68 + 4\lambda + 400\lambda + 300 = 0$ $\Rightarrow 413 + 413\lambda = 0$ $\Rightarrow \lambda = -1$ so S is (12, 32, -20)	B1 B1 M1 A1 A1 [5]	$\begin{pmatrix} 15 \\ 34 \\ 0 \end{pmatrix} + \dots$ $\dots + \lambda \begin{pmatrix} 3 \\ 2 \\ 20 \end{pmatrix}$ solving with plane $\lambda = -1$ cao

9(i) $v = \int 10e^{-\frac{1}{2}t} dt$ $= -20e^{-\frac{1}{2}t} + c$ when $t = 0$, $v = 0$ $\Rightarrow 0 = -20 + c$ $\Rightarrow c = 20$ so $v = 20 - 20e^{-\frac{1}{2}t}$	M1 A1 M1 A1 [4]	separate variables and intend to integrate $-20e^{-\frac{1}{2}t}$ finding c cao
(ii) As $t \rightarrow \infty$ $e^{-1/2t} \rightarrow 0$ $\Rightarrow v \rightarrow 20$ So long term speed is 20 m s^{-1}	M1 A1 [2]	ft (for their $c > 0$, found)
(iii) $\frac{1}{(w-4)(w+5)} = \frac{A}{w-4} + \frac{B}{w+5}$ $= \frac{A(w+5) + B(w-4)}{(w-4)(w+5)}$ $\Rightarrow 1 \equiv A(w+5) + B(w-4)$ $w = 4: 1 = 9A \Rightarrow A = 1/9$ $w = -5: 1 = -9B \Rightarrow B = -1/9$ $\Rightarrow \frac{1}{(w-4)(w+5)} = \frac{1/9}{w-4} - \frac{1/9}{w+5}$ $= \frac{1}{9(w-4)} - \frac{1}{9(w+5)}$	M1 M1 A1 A1 [4]	cover up, substitution or equating coeffs 1/9 -1/9
(iv) $\frac{dw}{dt} = -\frac{1}{2}(w-4)(w+5)$ $\Rightarrow \int \frac{dw}{(w-4)(w+5)} = \int -\frac{1}{2} dt$ $\Rightarrow \int \left[\frac{1}{9(w-4)} - \frac{1}{9(w+5)} \right] dw = \int -\frac{1}{2} dt$ $\Rightarrow \frac{1}{9} \ln(w-4) - \frac{1}{9} \ln(w+5) = -\frac{1}{2}t + c$ $\Rightarrow \frac{1}{9} \ln \frac{w-4}{w+5} = -\frac{1}{2}t + c$ When $t = 0$, $w = 10 \Rightarrow c = \frac{1}{9} \ln \frac{6}{15} = \frac{1}{9} \ln \frac{2}{5}$ $\Rightarrow \ln \frac{w-4}{w+5} = -\frac{9}{2}t + \ln \frac{2}{5}$ $\Rightarrow \frac{w-4}{w+5} = e^{-\frac{9}{2}t + \ln \frac{2}{5}} = \frac{2}{5} e^{-\frac{9}{2}t} = 0.4e^{-4.5t} *$	M1 M1 A1ft M1 M1 E1 [6]	separating variables substituting their partial fractions integrating correctly (condone absence of c) correctly evaluating c (at any stage) combining lns (at any stage) www
(v) As $t \rightarrow \infty$ $e^{-4.5t} \rightarrow 0$ $\Rightarrow w - 4 \rightarrow 0$ So long term speed is 4 m s^{-1}.	M1 A1 [2]	

Comprehension

1. (i)

2	1	3
3	2	1
1	3	2

B1
cao

(ii)

2	3	1
3	1	2
1	2	3

B1
cao

2. Dividing the grid up into four 2 x 2 blocks gives

1	2	3	4
3	1	4	2
2	4	1	3
4	3	2	1

Lines drawn on diagram or reference to 2 x 2 blocks. M1
One (or more) block does not contain all 4 of the symbols 1, 2, 3 and 4. oe. E1

3.

1	2	3	4
4	3	1	2
2	1	4	3
3	4	2	1

Many possible answers Row 2 correct B1
Rest correct B1

4. Either

4	2	3	1
		2	4
		4	2
2	4	1	3

Or

4	2	3	1
		2	4
		4	2
2	4	1	3

B2

5. In the top row there are 9 ways of allocating a symbol to the left cell, then 8 for the next, 7 for the next and so on down to 1 for the right cell, giving

M1

$9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 9!$ ways.

E1

So there must be $9!$ times the number of ways of completing the rest of the puzzle.

6.

(i)

Block side length, b	Sudoku, $s \times s$	M
1	1×1	-
2	4×4	12
3	9×9	77
4	16×16	252
5	25×25	621

25×25 B1

77, 252 and 621 B1

(ii)

$M = b^4 - 4$

b^4 B1

- 4 B1

7.

(i) There are neither 3s nor 5s among the givens. **M1**

So they are interchangeable and therefore there is no unique solution **E1**

(ii) The missing symbols form a 3×3 embedded Latin square. **M1**

There is not a unique arrangement of the numbers 1, 2 and 3 in this square. **E1**

[18]