

ADVANCED SUBSIDIARY GCE MATHEMATICS

4725/01

Further Pure Mathematics 1

MONDAY 2 JUNE 2008

Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)

List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are reminded of the need for clear presentation in your answers.

This document consists of 4 printed pages.

1 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix}$ and **I** is the 2×2 identity matrix. Find

(i)
$$A - 3I$$
, [2]

(ii)
$$A^{-1}$$
. [2]

- 2 The complex number 3 + 4i is denoted by a.
 - (i) Find |a| and arg a. [2]
 - (ii) Sketch on a single Argand diagram the loci given by

(a)
$$|z - a| = |a|$$
, [2]

(b)
$$arg(z-3) = arg a$$
. [3]

- 3 (i) Show that $\frac{1}{r!} \frac{1}{(r+1)!} = \frac{r}{(r+1)!}$. [2]
 - (ii) Hence find an expression, in terms of n, for

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!}.$$
 [4]

4 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$. Prove by induction that, for $n \ge 1$,

$$\mathbf{A}^{n} = \begin{pmatrix} 3^{n} & \frac{1}{2}(3^{n} - 1) \\ 0 & 1 \end{pmatrix}.$$
 [6]

- 5 Find $\sum_{r=1}^{n} r^2(r-1)$, expressing your answer in a fully factorised form. [6]
- 6 The cubic equation $x^3 + ax^2 + bx + c = 0$, where a, b and c are real, has roots (3 + i) and 2.
 - (i) Write down the other root of the equation. [1]
 - (ii) Find the values of a, b and c. [6]

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7	Describe fully	the geometrical	transformation 1	represented b	v each of the	following matrices:



(ii)
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
, [2]

(iii)
$$\begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$$
, [2]

(iv)
$$\begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}$$
. [2]

- 8 The quadratic equation $x^2 + kx + 2k = 0$, where k is a non-zero constant, has roots α and β . Find a quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [7]
- 9 (i) Use an algebraic method to find the square roots of the complex number 5 + 12i. [5]

(ii) Find
$$(3-2i)^2$$
. [2]

- (iii) Hence solve the quartic equation $x^4 10x^2 + 169 = 0$. [4]
- 10 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 8 & 10 \\ 2 & 1 & 2 \\ 4 & 3 & 6 \end{pmatrix}$. The matrix **B** is such that $\mathbf{AB} = \begin{pmatrix} a & 6 & 1 \\ 1 & 1 & 0 \\ 1 & 3 & 0 \end{pmatrix}$.
 - (i) Show that **AB** is non-singular. [2]
 - (ii) Find $(\mathbf{AB})^{-1}$. [4]
 - (iii) Find \mathbf{B}^{-1} . [5]

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