4730 Mechanics 3

1 i	Horiz. comp. of vel. after impact is 4ms^{-1} Vert. comp. of vel. after impact is $\sqrt{5^2 - 4^2} = 3\text{ms}^{-1}$ Coefficient of restitution is 0.5	B1 B1 B1 [3]	May be implied AG From e = 3/6
ii	Direction is vertically upwards Change of velocity is 3 – (-6) Impulse has magnitude 2.7Ns	B1 M1 A1 [3]	 From $m(\Delta v) = 0.3\times 9$
2 i	Horizontal component is 14N $80\times1.5 = 14\times1.5 + 3Y$ or $3(80 – Y) = 80\times1.5 + 14\times1.5$ or $1.5(80 – Y) = 14\times0.75 + 14\times0.75 + 1.5Y$ Vertical component is 33N upwards	B1 M1 A1 A1 [4]	 For taking moments for *AB* about *A* or *B* or the midpoint of *AB* AG
ii	Horizontal component at *C* is 14N [Vertical component at *C* is $(\pm)\sqrt{50^2 - 14^2}$] [$W = (\pm)48 - 33$] Weight is 15N	B1 M1 DM1 A1 [4]	May be implied for using $R^2 = H^2 + V^2$ For resolving forces at *C* vertically
3 i	 $4\times3\cos60^{o} – 2\times3\cos60^{o} = 2b$ $b = 1.5$ **j** component of vel. of *B* = $(-)3\sin60^{o}$ [$v^2 = b^2 + (-3\sin60^{o})^2$] Speed ($3\text{ms}^{-1}$) is unchanged [Angle with l.o.c. = $\tan^{-1}(3\sin60^{o}/1.5)$] Angle is 60^{o}.	M1 A1 A1 B1ft M1 A1ft M1 A1ft [8]	For using the p.c.mmtm parallel to l.o.c. ft consistent sin/cos mix For using $v^2 = b^2 + v_y^2$ AG ft - allow same answer following consistent sin/cos mix. For using angle = $\tan^{-1}(\pm v_y/v_x)$ ft consistent sin/cos mix
ii	[$e(3\cos60^{o} + 3\cos60^{o}) = 1.5$] Coefficient is 0.5	M1 A1ft [2]	For using NEL ft - allow same answer following consistent sin/cos mix throughout.

4 i	$F - 0.25v^2 = 120v(dv/dx)$ $F = 8000/v$ $[32000 - v^3 = 480v^2(dv/dx)]$ $\frac{480v^2}{v^3 - 32000}\frac{dv}{dx} = -1$	M1 A1 B1 M1 A1 [5]	For using Newton's second law with $a = v(dv/dx)$ For substituting for F and multiplying throughout by $4v$ (or equivalent) AG
ii	$\int \frac{480v^2}{v^3 - 32000}dv = -\int dx$ $160 \ln(v^3 - 32000) = -x \quad (+A)$ $160 \ln(v^3 - 32000) = -x + 160 \ln 32000$ or $160 \ln(v^3 - 32000) - 160 \ln 32000 = -500$ $(v^3 - 32000)/32000 = e^{-x/160}$ Speed of m/c is $32.2 ms^{-1}$	M1 A1 M1 A1ft B1ft B1 [6]	For separating variables and integrating For using $v(0) = 40$ or $[160 \ln(v^3 - 32000)]_{40}^{v} = [-x]_0^{500}$ ft where factor 160 is incorrect but +ve, Implied by $(v^3 - 32000)/32000 = e^{-3.125}$ (or = 0.0439 ..). ft where factor 160 is incorrect but +ve, or for an incorrect non-zero value of A
5 i	$x_{max} = \sqrt{1.5^2 + 2^2} - 1.5 \ (= 1)$ $[T_{max} = 18\times1/1.5]$ Maximum tension is 12N	B1 M1 A1 [3]	 For using $T = \lambda x/L$
      ii	**(a)** Gain in EE = $2[18(1^2 - 0.2^2)]/(2\times1.5)$ (11.52) Loss in GPE = 2.8mg (27.44m) $[2.8m \times 9.8 = 11.52]$ $m = 0.42$ **(b)** $\frac{1}{2} mv^2 = mg(0.8) + 2\times18\times0.2^2/(2\times1.5)$ or $\frac{1}{2} mv^2 = 2\times18\times1^2/(2\times1.5) - mg(2)$ Speed at M is $4.24 ms^{-1}$	M1 A1 B1 M1 A1 [5] M1 A1ft A1ft [3]	For using EE = $\lambda x^2/2L$ May be scored with correct EE terms in expressions for total energy on release and total energy at lowest point May be scored with correct GPE terms in expressions for total energy on release and total energy at lowest point For using the p.c.energy AG For using the p.c.energy KE, PE & EE must all be represented ft only when just one string is considered throughout in evaluating EE ft only for answer 4.10 following consideration of only one string

6 **i**	[$-mg\sin\theta = mL(d^2\theta/dt^2)$] $d^2\theta/dt^2 = -(g/L)\sin\theta$	M1 A1 [2]	For using Newton's second law tangentially with $a = Ld^2\theta/dt^2$ AG
ii	[$d^2\theta/dt^2 = -(g/L)\ \theta$] $d^2\theta/dt^2 = -(g/L)\ \theta$ ➔ motion is SH	M1 A1 [2]	For using $\sin\theta \approx \theta$ because θ is small ($\theta_{max} = 0.05$) AG
iii	[$4\pi/7 = 2\pi/\sqrt{9.8/L}$] $L = 0.8$	M1 A1 [2]	For using $T = 2\pi/n$ where $-n^2$ is coefficient of θ
iv	[$\theta = 0.05\cos 3.5\times 0.7$] $\theta = -0.0385$ $t = 1.10$ (accept 1.1 or 1.09)	M1 A1ft M1 A1ft [4]	For using $\theta = \theta_o\cos nt$ {$\theta = \theta_o\sin nt$ not accepted unless the t is reconciled with the t as defined in the question} ft incorrect L {$\theta = 0.05\cos[4.9/(5L)^{½}]$} For attempting to find 3.5t ($\pi < 3.5t < 1.5\pi$) for which $0.05\cos 3.5t$ = answer found for θ or for using $3.5(t_1 + t_2) = 2\pi$ ft incorrect L {$t = [2\pi(5L)^{½}]/7 - 0.7$}
v	$\dot{\theta}^2 = 3.5^2(0.05^2 - (-0.0385)^2)$ or $\dot{\theta} = -3.5\times 0.05\sin(3.5\times 0.7)$ ($\dot{\theta} = -0.1116..$) Speed is 0.0893ms^{-1} (Accept answers correct to 2 s.f.)	M1 A1ft A1ft [3]	For using $\dot{\theta}^2 = n^2(\theta_o^2 - \theta^2)$ or $\dot{\theta} = -n\ \theta_o\sin nt$ {also allow $\dot{\theta} = n\theta_o\cos nt$ if $\theta = \theta_o\sin nt$ has been used previously} ft incorrect θ with or without 3.5 represented by $(g/L)^{½}$ using incorrect L in (iii) or for $\dot{\theta} = 3.5\times 0.05\cos(3.5\times 0.7)$ following previous use of $\theta = \theta_o\sin nt$ ft incorrect L ($L\times 0.089287/0.8$ with $n = 3.5$ used or from $\lvert 0.35\sin\{4.9/[5L]^{½}\}/[5L]^{½}\rvert$ --- **SR** for candidates who use $\dot{\theta}$ as v. (Max 1/3) For v = ± 0.112 B1

7 i	Gain in PE = $mga(1 - \cos\theta)$ [$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mga(1 - \cos\theta)$]	B1 M1	 For using KE loss = PE gain
	$v^2 = u^2 - 2ga(1 - \cos\theta)$ [$R - mg\cos\theta = m$(accel.)] $R = mv^2/a + mg\cos\theta$ [$R = m\{u^2 - 2ga(1 - \cos\theta)\}/a + mg\cos\theta$] $R = mu^2/a + mg(3\cos\theta - 2)$	A1 M1 A1 M1 A1 [7]	 For using Newton's second law radially For substituting for v^2 AG
ii	[$0 = mu^2/a - 5mg$] $u^2 = 5ag$ [$v^2 = 5ag - 4ag$] Least value of v^2 is ag	M1 A1 M1 A1 [4]	For substituting $R = 0$ and $\theta = 180^o$ For substituting for u^2 (= $5ag$) and $\theta = 180^o$ in v^2 (expression found in (i)) { but M0 if $v = 0$ has been used to find u^2} AG
iii	 [$0 = u^2 - 2ga(1 - \sqrt{3}/2)$] $u^2 = ag(2 - \sqrt{3})$	 M1 A1 [2]	For substituting $v^2 = 0$ and $\theta = \pi/6$ in v^2 (expression found in (i)) Accept $u^2 = 2ag(1 - \cos\pi/6)$