OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4753/01

Methods for Advanced Mathematics (C3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 5 June 2009
Afternoon

Duration: 1 hour 30 minutes

4753O1

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Evaluate $\int_0^{\frac{1}{6}\pi} \sin 3x \, dx$. **[3]**

2 A radioactive substance decays exponentially, so that its mass M grams can be modelled by the equation $M = Ae^{-kt}$, where t is the time in years, and A and k are positive constants.

(i) An initial mass of 100 grams of the substance decays to 50 grams in 1500 years. Find A and k. **[5]**

(ii) The substance becomes safe when 99% of its initial mass has decayed. Find how long it will take before the substance becomes safe. **[3]**

3 Sketch the curve $y = 2 \arccos x$ for $-1 \leqslant x \leqslant 1$. **[3]**

4 Fig. 4 shows a sketch of the graph of $y = 2|x - 1|$. It meets the x- and y-axes at $(a, 0)$ and $(0, b)$ respectively.

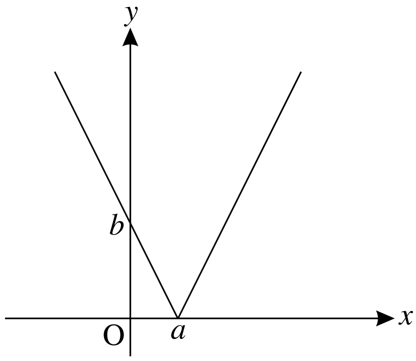

Fig. 4

Find the values of a and b. **[3]**

5 The equation of a curve is given by $e^{2y} = 1 + \sin x$.

(i) By differentiating implicitly, find $\frac{dy}{dx}$ in terms of x and y. **[3]**

(ii) Find an expression for y in terms of x, and differentiate it to verify the result in part **(i)**. **[4]**

6 Given that $f(x) = \frac{x+1}{x-1}$, show that $ff(x) = x$.

Hence write down the inverse function $f^{-1}(x)$. What can you deduce about the symmetry of the curve $y = f(x)$? **[5]**

7 **(i)** Show that

(*A*) $(x-y)(x^2+xy+y^2) = x^3 - y^3$,

(*B*) $(x+\frac{1}{2}y)^2 + \frac{3}{4}y^2 = x^2 + xy + y^2$. **[4]**

(ii) Hence prove that, for all real numbers x and y, if $x > y$ then $x^3 > y^3$. **[3]**

Section B (36 marks)

8 Fig. 8 shows the line $y = x$ and parts of the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$, where

$$\mathrm{f}(x) = \mathrm{e}^{x-1}, \quad \mathrm{g}(x) = 1 + \ln x.$$

The curves intersect the axes at the points A and B, as shown. The curves and the line $y = x$ meet at the point C.

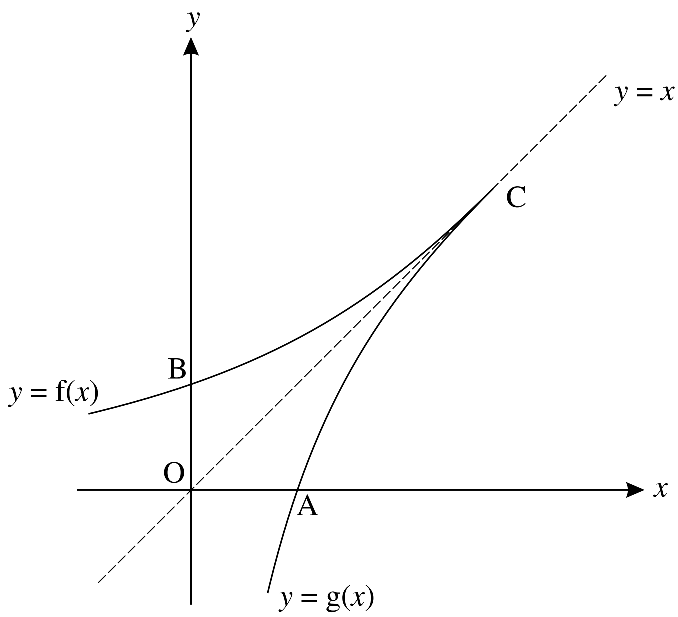

Fig. 8

(i) Find the exact coordinates of A and B. Verify that the coordinates of C are (1, 1). **[5]**

(ii) Prove algebraically that $\mathrm{g}(x)$ is the inverse of $\mathrm{f}(x)$. **[2]**

(iii) Evaluate $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in terms of e. **[3]**

(iv) Use integration by parts to find $\int \ln x \,\mathrm{d}x$.

Hence show that $\int_{\mathrm{e}^{-1}}^{1} \mathrm{g}(x)\,\mathrm{d}x = \frac{1}{\mathrm{e}}$. **[6]**

(v) Find the area of the region enclosed by the lines OA and OB, and the arcs AC and BC. **[2]**

9 Fig. 9 shows the curve $y = \dfrac{x^2}{3x - 1}$.

P is a turning point, and the curve has a vertical asymptote $x = a$.

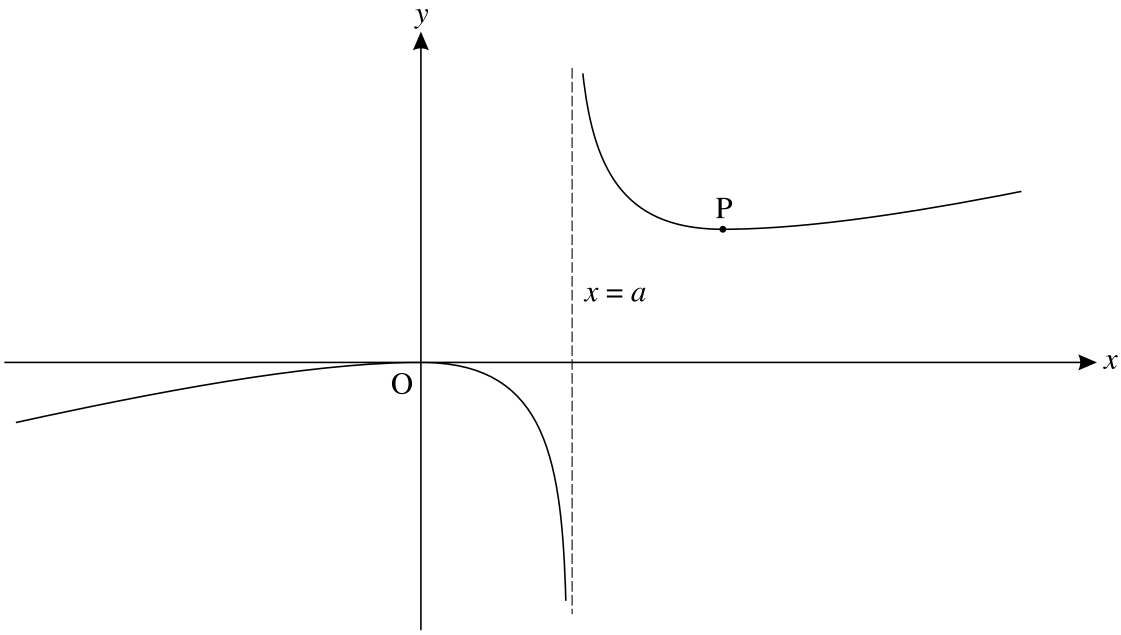

Fig. 9

(i) Write down the value of a. **[1]**

(ii) Show that $\dfrac{dy}{dx} = \dfrac{x(3x - 2)}{(3x - 1)^2}$. **[3]**

(iii) Find the exact coordinates of the turning point P.

Calculate the gradient of the curve when $x = 0.6$ and $x = 0.8$, and hence verify that P is a minimum point. **[7]**

(iv) Using the substitution $u = 3x - 1$, show that $\displaystyle\int \frac{x^2}{3x - 1}\,dx = \frac{1}{27}\int \left(u + 2 + \frac{1}{u}\right) du$.

Hence find the exact area of the region enclosed by the curve, the x-axis and the lines $x = \frac{2}{3}$ and $x = 1$. **[7]**