

**ADVANCED GCE**

**MATHEMATICS (MEI)**

Methods for Advanced Mathematics (C3)

**4753/01**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**

None

**Friday 5 June 2009**  
**Afternoon**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

## Section A (36 marks)

- 1 Evaluate  $\int_0^{\frac{1}{6}\pi} \sin 3x \, dx$ . [3]
- 2 A radioactive substance decays exponentially, so that its mass  $M$  grams can be modelled by the equation  $M = Ae^{-kt}$ , where  $t$  is the time in years, and  $A$  and  $k$  are positive constants.
- (i) An initial mass of 100 grams of the substance decays to 50 grams in 1500 years. Find  $A$  and  $k$ . [5]
- (ii) The substance becomes safe when 99% of its initial mass has decayed. Find how long it will take before the substance becomes safe. [3]
- 3 Sketch the curve  $y = 2 \arccos x$  for  $-1 \leq x \leq 1$ . [3]
- 4 Fig. 4 shows a sketch of the graph of  $y = 2|x - 1|$ . It meets the  $x$ - and  $y$ -axes at  $(a, 0)$  and  $(0, b)$  respectively.

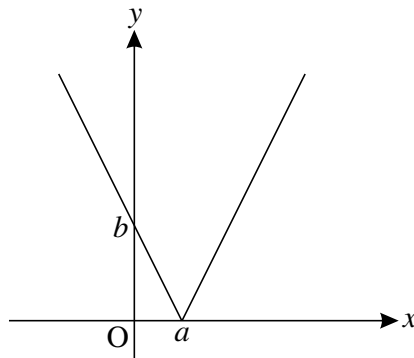


Fig. 4

- Find the values of  $a$  and  $b$ . [3]
- 5 The equation of a curve is given by  $e^{2y} = 1 + \sin x$ .
- (i) By differentiating implicitly, find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ . [3]
- (ii) Find an expression for  $y$  in terms of  $x$ , and differentiate it to verify the result in part (i). [4]
- 6 Given that  $f(x) = \frac{x+1}{x-1}$ , show that  $ff(x) = x$ .

Hence write down the inverse function  $f^{-1}(x)$ . What can you deduce about the symmetry of the curve  $y = f(x)$ ? [5]

7 (i) Show that

$$(A) \quad (x - y)(x^2 + xy + y^2) = x^3 - y^3,$$

$$(B) \quad \left(x + \frac{1}{2}y\right)^2 + \frac{3}{4}y^2 = x^2 + xy + y^2. \quad [4]$$

(ii) Hence prove that, for all real numbers  $x$  and  $y$ , if  $x > y$  then  $x^3 > y^3$ . [3]

### Section B (36 marks)

8 Fig. 8 shows the line  $y = x$  and parts of the curves  $y = f(x)$  and  $y = g(x)$ , where

$$f(x) = e^{x-1}, \quad g(x) = 1 + \ln x.$$

The curves intersect the axes at the points A and B, as shown. The curves and the line  $y = x$  meet at the point C.

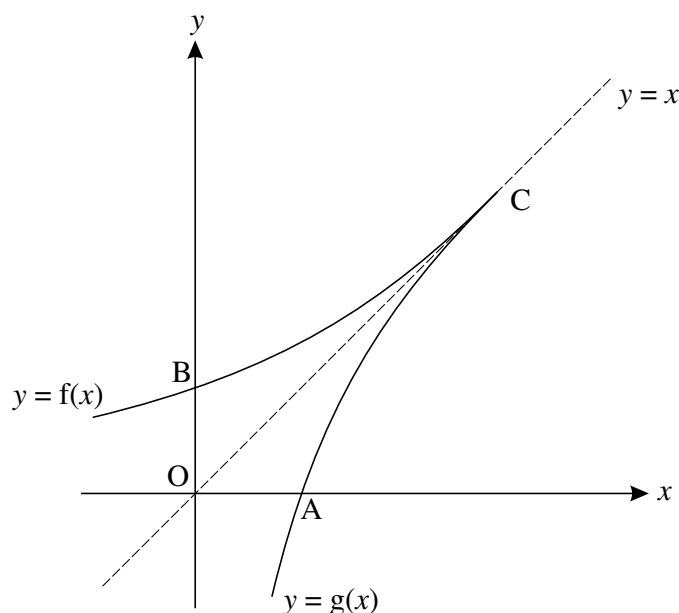


Fig. 8

(i) Find the exact coordinates of A and B. Verify that the coordinates of C are (1, 1). [5]

(ii) Prove algebraically that  $g(x)$  is the inverse of  $f(x)$ . [2]

(iii) Evaluate  $\int_0^1 f(x) \, dx$ , giving your answer in terms of  $e$ . [3]

(iv) Use integration by parts to find  $\int \ln x \, dx$ .

Hence show that  $\int_{e^{-1}}^1 g(x) \, dx = \frac{1}{e}$ . [6]

(v) Find the area of the region enclosed by the lines OA and OB, and the arcs AC and BC. [2]

- 9 Fig. 9 shows the curve  $y = \frac{x^2}{3x-1}$ .

P is a turning point, and the curve has a vertical asymptote  $x = a$ .

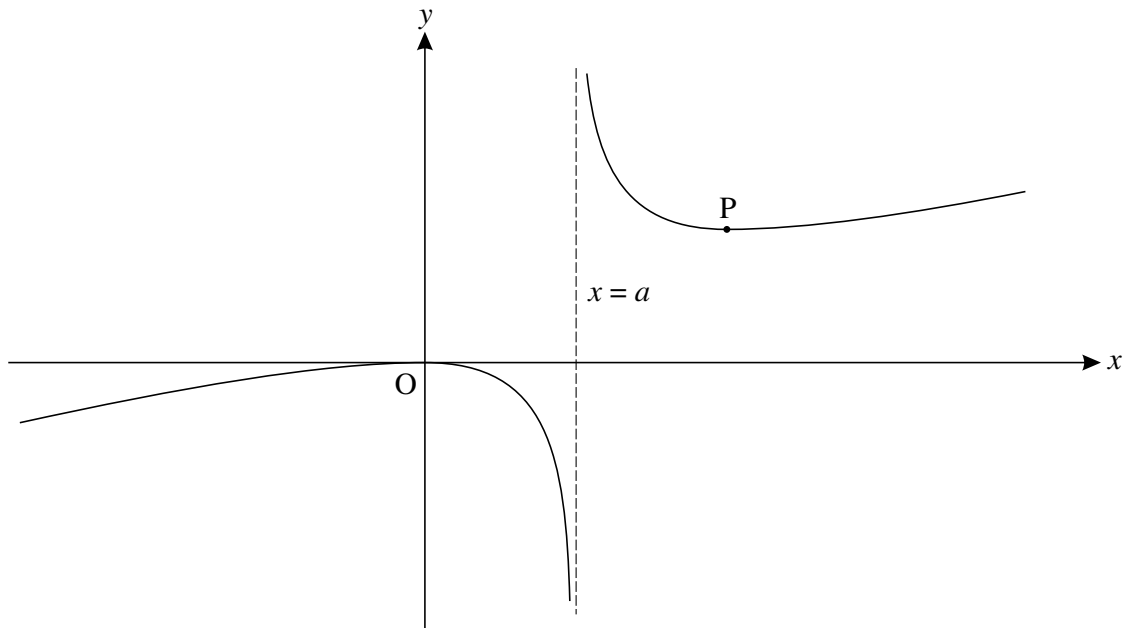


Fig. 9

- (i) Write down the value of  $a$ . [1]
- (ii) Show that  $\frac{dy}{dx} = \frac{x(3x-2)}{(3x-1)^2}$ . [3]
- (iii) Find the exact coordinates of the turning point P.

Calculate the gradient of the curve when  $x = 0.6$  and  $x = 0.8$ , and hence verify that P is a minimum point. [7]

- (iv) Using the substitution  $u = 3x - 1$ , show that  $\int \frac{x^2}{3x-1} dx = \frac{1}{27} \int \left(u + 2 + \frac{1}{u}\right) du$ .

Hence find the exact area of the region enclosed by the curve, the  $x$ -axis and the lines  $x = \frac{2}{3}$  and  $x = 1$ . [7]

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