

# ADVANCED SUBSIDIARY GCE MATHEMATICS

Further Pure Mathematics 1

4725

Candidates answer on the Answer Booklet

#### **OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

## **Other Materials Required:**

None

Friday 5 June 2009 Afternoon

Duration: 1 hour 30 minutes



#### **INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer all the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

## **INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [] at the end of each question or part question.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- This document consists of 4 pages. Any blank pages are indicated.

1 Evaluate 
$$\sum_{r=101}^{250} r^3$$
. [3]

- 2 The matrices **A** and **B** are given by  $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$  and **I** is the 2 × 2 identity matrix. Find the values of the constants a and b for which  $a\mathbf{A} + b\mathbf{B} = \mathbf{I}$ .
- 3 The complex numbers z and w are given by z = 5 2i and w = 3 + 7i. Giving your answers in the form x + iy and showing clearly how you obtain them, find

(i) 
$$4z - 3w$$
, [2]

(ii) 
$$z^*w$$
. [2]

- 4 The roots of the quadratic equation  $x^2 + x 8 = 0$  are p and q. Find the value of  $p + q + \frac{1}{p} + \frac{1}{q}$ . [4]
- 5 The cubic equation  $x^3 + 5x^2 + 7 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .
  - (i) Use the substitution  $x = \sqrt{u}$  to find a cubic equation in u with integer coefficients. [3]
  - (ii) Hence find the value of  $\alpha^2 \beta^2 + \beta^2 \gamma^2 + \gamma^2 \alpha^2$ . [2]
- 6 The complex number 3 3i is denoted by a.
  - (i) Find |a| and arg a. [2]
  - (ii) Sketch on a single Argand diagram the loci given by

(a) 
$$|z-a| = 3\sqrt{2}$$
, [3]

**(b)** 
$$\arg(z-a) = \frac{1}{4}\pi$$
. [3]

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z-a| \ge 3\sqrt{2}$$
 and  $0 \le \arg(z-a) \le \frac{1}{4}\pi$ . [3]

7 (i) Use the method of differences to show that

$$\sum_{r=1}^{n} \{ (r+1)^4 - r^4 \} = (n+1)^4 - 1.$$
 [2]

(ii) Show that 
$$(r+1)^4 - r^4 = 4r^3 + 6r^2 + 4r + 1$$
. [2]

(iii) Hence show that

$$4\sum_{r=1}^{n}r^{3}=n^{2}(n+1)^{2}.$$
 [6]

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- 8 The matrix C is given by  $C = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ .
  - (i) Draw a diagram showing the image of the unit square under the transformation represented by C. [3]

The transformation represented by C is equivalent to a transformation S followed by another transformation T.

- (ii) Given that S is a shear with the y-axis invariant in which the image of the point (1, 1) is (1, 2), write down the matrix that represents S. [2]
- (iii) Find the matrix that represents transformation T and describe fully the transformation T. [6]
- 9 The matrix **A** is given by  $\mathbf{A} = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 2 \end{pmatrix}$ .
  - (i) Find, in terms of a, the determinant of A. [3]
  - (ii) Hence find the values of a for which A is singular. [3]
  - (iii) State, giving a brief reason in each case, whether the simultaneous equations

$$ax + y + z = 2a$$
,  
 $x + ay + z = -1$ ,  
 $x + y + 2z = -1$ ,

have any solutions when

- (a) a = 0,
- **(b)** a = 1.

[4]

- **10** The sequence  $u_1, u_2, u_3, ...$  is defined by  $u_1 = 3$  and  $u_{n+1} = 3u_n 2$ .
  - (i) Find  $u_2$  and  $u_3$  and verify that  $\frac{1}{2}(u_4 1) = 27$ . [3]
  - (ii) Hence suggest an expression for  $u_n$ . [2]
  - (iii) Use induction to prove that your answer to part (ii) is correct. [5]

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