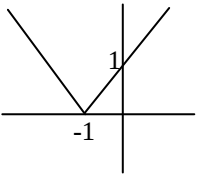
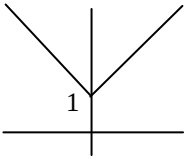


Mathematics (MEI)

Advanced GCE **4753**

Methods for Advanced Mathematics (C3)

Mark Scheme for June 2010

1 $\int_0^{\pi/6} \cos 3x \, dx = \left[\frac{1}{3} \sin 3x \right]_0^{\pi/6}$ $= \frac{1}{3} \sin \frac{\pi}{2} - 0$ $= 1/3$	M1 B1 A1cao [3]	$k \sin 3x, k > 0, k \neq 3$ $k = (\pm)1/3$ 0.33 or better	or M1 for $u = 3x \Rightarrow \int \frac{1}{3} \cos u \, du$ condone 90° in limit or M1 for $\left[\frac{1}{3} \sin u \right]$ so: $\sin 3x$: M1B0, $-\sin 3x$: M0B0, $\pm 3 \sin 3x$: M0B0, $-1/3 \sin 3x$: M0B1
2 $fg(x) = x+1 $ $gf(x) = x +1$  	B1 B1 B1 B1 [4]	soi from correctly-shaped graphs (i.e. without intercepts) graph of $ x+1 $ only graph of $ x +1$	but must indicate which is which bod gf if negative x values are missing ‘V’ shape with $(-1, 0)$ and $(0, 1)$ labelled ‘V’ shape with $(0, 1)$ labelled $(0, 1)$
3(i) $y = (1+3x^2)^{1/2}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{2}(1+3x^2)^{-1/2} \cdot 6x$ $= 3x(1+3x^2)^{-1/2}$	M1 B1 A1 [3]	chain rule $\frac{1}{2} u^{-1/2}$ o.e., but must be ‘3’	can isw here
(ii) $y = x(1+3x^2)^{1/2}$ $\Rightarrow \frac{dy}{dx} = x \cdot \frac{3x}{\sqrt{1+3x^2}} + 1 \cdot (1+3x^2)^{1/2}$ $= \frac{3x^2 + 1 + 3x^2}{\sqrt{1+3x^2}}$ $= \frac{1+6x^2}{\sqrt{1+3x^2}} *$	M1 A1ft M1 E1 [4]	product rule ft their dy/dx from (i) common denominator or factoring $(1+3x^2)^{-1/2}$ www	must show this step for M1 E1

4 $p = 100/x = 100 x^{-1}$ $\Rightarrow dp/dx = -100x^{-2} = -100/x^2$ $dp/dt = dp/dx \times dx/dt$ $dx/dt = 10$ When $x = 50$, $dp/dx = (-100/50^2)$ $\Rightarrow dp/dt = 10 \times -0.04 = -0.4$	M1 A1 M1 B1 M1dep A1cao [6]	attempt to differentiate $-100x^{-2}$ o.e. o.e. soi soi substituting $x = 50$ into their dp/dx dep 2nd M1 o.e. e.g. decreasing at 0.4	condone poor notation if chain rule correct or $x = 50 + 10 t$ B1 $\Rightarrow P = 100/x = 100/(50 + 10 t)$ $\Rightarrow dP/dt = -100(50 + 10 t)^{-2} \times 10 = -1000/(50 + 10t)^{-2}$ M1 A1 When $t = 0$, $dP/dt = -1000/50^2 = -0.4$ A1
5 $y^3 = xy - x^2$ $\Rightarrow 3y^2 dy/dx = x dy/dx + y - 2x$ $\Rightarrow 3y^2 dy/dx - x dy/dx = y - 2x$ $\Rightarrow (3y^2 - x) dy/dx = y - 2x$ $\Rightarrow dy/dx = (y - 2x)/(3y^2 - x) *$ TP when $dy/dx = 0 \Rightarrow y - 2x = 0$ $\Rightarrow y = 2x$ $\Rightarrow (2x)^3 = x \cdot 2x - x^2$ $\Rightarrow 8x^3 = x^2$ $\Rightarrow x = 1/8 *(or 0)$	B1 B1 M1 E1 M1 M1 E1 [7]	$3y^2 dy/dx$ $x dy/dx + y - 2x$ collecting terms in dy/dx only or $x = 1/8$ and $dy/dx = 0 \Rightarrow y = 1/4$ or $(1/4)^3 = (1/8)(1/4) - (1/8)^2$ or verifying e.g. $1/64 = 1/64$	must show '$x dy/dx + y$' on one side or $x = 1/8 \Rightarrow y^3 = (1/8)y - 1/64$ M1 verifying that $y = 1/4$ is a solution (must show evidence*) M1 $\Rightarrow dy/dx = (1/4 - 2(1/8))/(...) = 0$ E1 *just stating that $y = 1/4$ is M1 M0 E0
6 $f(x) = 1 + 2 \sin 3x = y \quad x \leftrightarrow y$ $x = 1 + 2 \sin 3y$ $\Rightarrow \sin 3y = (x - 1)/2$ $\Rightarrow 3y = \arcsin [(x - 1)/2]$ $\Rightarrow y = \frac{1}{3} \arcsin \left[\frac{x-1}{2} \right]$ so $f^{-1}(x) = \frac{1}{3} \arcsin \left[\frac{x-1}{2} \right]$ Range of f is -1 to 3 $\Rightarrow -1 \leq x \leq 3$	M1 A1 A1 A1 M1 A1 [6]	attempt to invert must be $y = \dots$ or $f^{-1}(x) = \dots$ or $-1 \leq (x - 1)/2 \leq 1$ must be 'x', not y or $f(x)$	at least one step attempted, or reasonable attempt at flow chart inversion (or any other variable provided same used on each side) condone '<'s for M1 allow unsupported correct answers; -1 to 3 is M1 A0
7 (A) True , (B) True , (C) False Counterexample, e.g. $\sqrt{2} + (-\sqrt{2}) = 0$	B2,1,0 B1 [3]		

8(i) When $x = 1$, $y = 3 \ln 1 + 1 - 1^2 = 0$	E1 [1]		
(ii) $\frac{dy}{dx} = \frac{3}{x} + 1 - 2x$ $\Rightarrow$ At R, $\frac{dy}{dx} = 0 = \frac{3}{x} + 1 - 2x$ $\Rightarrow 3 + x - 2x^2 = 0$ $\Rightarrow (3 - 2x)(1 + x) = 0$ $\Rightarrow x = 1.5$, (or -1) $\Rightarrow y = 3 \ln 1.5 + 1.5 - 1.5^2 = 0.466$ (3 s.f.) $\frac{d^2y}{dx^2} = -\frac{3}{x^2} - 2$ When $x = 1.5$, $d^2y/dx^2 (= -10/3) < 0 \Rightarrow \text{max}$	M1 A1cao M1 M1 A1 M1 A1cao B1ft E1 [9]	$d/dx (\ln x) = 1/x$ re-arranging into a quadratic = 0 factorising or formula or completing square substituting their x ft their dy/dx on equivalent work www – don't need to calculate 10/3	 SC1 for $x = 1.5$ unsupported, SC3 if verified but condone rounding errors on 0.466
(iii) Let $u = \ln x$, $du/dx = 1/x$ $dv/dx = 1$, $v = x$ $\Rightarrow \int \ln x dx = x \ln x - \int x \cdot \frac{1}{x} dx$ $= x \ln x - \int 1 \cdot dx$ $= x \ln x - x + c$ $\Rightarrow A = \int_1^{2.05} (3 \ln x + x - x^2) dx$ $= \left[3x \ln x - 3x + \frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_1^{2.05}$ $= -2.5057 + 2.833..$ $= 0.33$ (2 s.f.)	M1 A1 A1 B1 B1ft M1dep A1 cao [7]	parts condone no c correct integral and limits (soi) $\left[3 \times \text{their 'x ln x - x' + } \frac{1}{2}x^2 - \frac{1}{3}x^3 \right]$ substituting correct limits dep 1 st B1	 allow correct result to be quoted (SC3)

9(i) $(0, \frac{1}{2})$	B1 [1]	allow $y = \frac{1}{2}$, but not $(x =) \frac{1}{2}$ or $(\frac{1}{2}, 0)$ nor $P = 1/2$	
(ii) $\frac{dy}{dx} = \frac{(1+e^{2x})2e^{2x} - e^{2x} \cdot 2e^{2x}}{(1+e^{2x})^2}$ $= \frac{2e^{2x}}{(1+e^{2x})^2}$ When $x = 0$, $dy/dx = 2e^0/(1+e^0)^2 = \frac{1}{2}$	M1 A1 A1 B1ft [4]	Quotient or product rule correct expression – condone missing bracket cao – mark final answer follow through their derivative	product rule: $\frac{dy}{dx} = e^{2x} \cdot 2e^{2x}(-1)(1+e^{2x})^{-2} + 2e^{2x}(1+e^{2x})^{-1}$ $-\frac{2e^{2x}}{(1+e^{2x})^2}$ from $(udv - vdu)/v^2$ SC1
(iii) $A = \int_0^1 \frac{e^{2x}}{1+e^{2x}} dx$ $= \left[\frac{1}{2} \ln(1+e^{2x}) \right]_0^1$ or let $u = 1 + e^{2x}$, $du/dx = 2e^{2x}$ $\Rightarrow A = \int_2^{1+e^2} \frac{1/2}{u} du = \left[\frac{1}{2} \ln u \right]_2^{1+e^2}$ $= \frac{1}{2} \ln(1+e^2) - \frac{1}{2} \ln 2$ $= \frac{1}{2} \ln \left[\frac{1+e^2}{2} \right]^*$	B1 M1 A1 M1 A1 M1 E1 [5]	correct integral and limits (soi) $k \ln(1+e^{2x})$ $k = \frac{1}{2}$ or $v = e^{2x}$, $dv/dx = 2e^{2x}$ o.e. [$\frac{1}{2} \ln u$] or [$\frac{1}{2} \ln(v+1)$] substituting correct limits www	condone no dx allow missing dx's or incompatible limits, but penalise missing brackets
(iv) $g(-x) = \frac{1}{2} \left[\frac{e^{-x} - e^x}{e^{-x} + e^x} \right] = -\frac{1}{2} \left[\frac{e^x - e^{-x}}{e^x + e^{-x}} \right] = -g(x)$ Rotational symmetry of order 2 about O	M1 E1 B1 [3]	substituting $-x$ for x in $g(x)$ completion www – taking out $-ve$ must be clear must have ‘rotational’ ‘about O’, ‘order 2’ (oe)	not $g(-x) \neq g(x)$. Condone use of f for g.
(v)(A) $g(x) + \frac{1}{2} = \frac{1}{2} \cdot \frac{e^x - e^{-x}}{e^x + e^{-x}} + \frac{1}{2} = \frac{1}{2} \cdot \left(\frac{e^x - e^{-x} + e^x + e^{-x}}{e^x + e^{-x}} \right)$ $= \frac{1}{2} \cdot \left(\frac{2e^x}{e^x + e^{-x}} \right)$ $= \frac{e^x \cdot e^x}{e^x(e^x + e^{-x})} = \frac{e^{2x}}{e^{2x} + 1} = f(x)$ (B) Translation $\begin{pmatrix} 0 \\ 1/2 \end{pmatrix}$ (C) Rotational symmetry [of order 2] about P	M1 A1 E1 M1 A1 B1 [6]	combining fractions (correctly) translation in y direction up $\frac{1}{2}$ unit dep ‘translation’ used o.e. condone omission of 180° /order 2	allow ‘shift’, ‘move’ in correct direction for M1. $\begin{pmatrix} 0 \\ 1/2 \end{pmatrix}$ alone is SC1.