GCE

Mathematics (MEI)

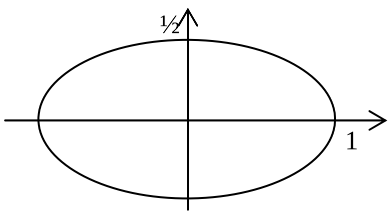
Advanced GCE **4754A**

Applications of Advanced Mathematics (C4) Paper A

Mark Scheme for June 2010

Section A

1 $\frac{x}{x^2-1}+\frac{2}{x+1}=\frac{x}{(x-1)(x+1)}+\frac{2}{x+1}$	B1	$x^2 - 1 = (x + 1)(x - 1)$
$=\frac{x+2(x-1)}{(x-1)(x+1)}$	M1	correct method for addition of fractions
$=\frac{(3x-2)}{(x-1)(x+1)}$	A1	or $\frac{(3x-2)}{x^2-1}$ do not isw for incorrect subsequent cancelling
or $\frac{x}{x^2-1}+\frac{2}{x+1}=\frac{x(x+1)+2(x^2-1)}{(x^2-1)(x+1)}$	M1	correct method for addition of fractions
$=\frac{3x^2+x-2}{(x^2-1)(x+1)}$		
$=\frac{(3x-2)(x+1)}{(x^2-1)(x+1)}$	B1	$(3x - 2)(x + 1)$
$=\frac{(3x-2)}{(x^2-1)}$	A1 [3]	accept denominator as x^2-1 or(x-1)(x+1) do not isw for incorrect subsequent cancelling
2(i) When $x = 0.5$, $y = 1.1180$	B1	4dp
$\Rightarrow$ $A \approx 0.25/2\{1+1.4142+2(1.0308+1.1180+1.25)\}$	M1	
$= 0.25 \times 4.6059 = 1.151475$		(0.125 x 9.2118)
$= 1.151$ (3 d.p.)*	E1 [3]	need evidence
(ii) Explain that the area is an over-estimate. *or* The curve is below the trapezia, so the area is an over- estimate.	B1	or use a diagram to show why
This becomes less with more strips. *or* Greater number of strips improves accuracy so becomes less	B1 [2]	
(iii) $V = \int_0^1 \pi y^2 dx$		
$= \int_0^1 \pi(1+x^2)dx$	M1	allow limits later
$= \pi\left[(x+x^3/3)\right]_0^1$	B1	$x + x^3/3$
$= 1\frac{1}{3}\pi$	A1 [3]	exact

3 $y = \sin\theta\cos\theta = ½\sin 2\theta$ $x = \cos 2\theta$ $\sin^2 2\theta + \cos^2 2\theta = 1$ $\Rightarrow x^2 + (2y)^2 = 1$ $\Rightarrow x^2 + 4y^2 = 1$ *	M1 M1 E1	use of sin 2θ
or $x^2 + 4y^2 = (\cos 2\theta)^2 + 4(\sin\theta\cos\theta)^2$ $= \cos^2 2\theta + \sin^2 2\theta$ $= 1$ *	M1 M1 E1	substitution use of sin2θ
or $\cos 2\theta = 2\cos^2\theta - 1$ $\cos^2\theta = (x+1)/2$ $\cos 2\theta = 1 - 2\sin^2\theta$ $\sin^2\theta = (1-x)/2$ $y^2 = \sin^2\theta\cos^2\theta = \left(\frac{1-x}{2}\right)\left(\frac{x+1}{2}\right)$ $y^2 = (1-x^2)/4$ $x^2 + 4y^2 = 1$*	 M1 M1 E1	 for both
or $x = \cos 2\theta = \cos^2\theta - \sin^2\theta$ $x^2 = \cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta$ $y^2 = \sin^2\theta\cos^2\theta$ $x^2 + 4y^2 = \cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta + 4\sin^2\theta\cos^2\theta$ $= (\cos^2\theta + \sin^2\theta)^2$ $= 1$*	 M1 M1 E1	 correct use of double angle formulae correct squaring and use of $\sin^2\theta + \cos^2\theta = 1$
	M1 A1 [5]	ellipse correct intercepts

4 $\sqrt{4+x} = 2(1+\frac{x}{4})^{\frac{1}{2}}$ $= 2(1+\frac{1}{2}.\frac{x}{4}+\frac{\frac{1}{2}.-\frac{1}{2}}{2}(\frac{x}{4})^2+...)$ $= 2(1+\frac{1}{8}x-\frac{1}{128}x^2+...)$ $= 2+\frac{1}{4}x-\frac{1}{64}x^2+...$ Valid for $-1 < x/4 < 1$ $\Rightarrow -4 < x < 4$	M1 M1 A1 A1 B1 [5]	dealing with √4 (or terms in $4^{\frac{1}{2}}$, $4^{\frac{-1}{2}}$,…etc) correct binomial coefficients correct **unsimplified** expression for $(1+x/4)^{\frac{1}{2}}$ or $(4+x)^{\frac{1}{2}}$ cao
5(i) $\frac{3}{(y-2)(y+1)} = \frac{A}{y-2}+\frac{B}{y+1}$ $= \frac{A(y+1)+B(y-2)}{(y-2)(y+1)}$ $\Rightarrow 3 = A(y+1) + B(y-2)$ $y = 2 \Rightarrow 3 = 3A \Rightarrow A = 1$ $y = -1 \Rightarrow 3 = -3B \Rightarrow B = -1$	 M1 A1 A1 [3]	 substituting, equating coeffs or cover up
(ii) $\frac{dy}{dx} = x^2(y-2)(y+1)$ $\Rightarrow \int \frac{3\,dy}{(y-2)(y+1)} = \int 3x^2\,dx$ $\Rightarrow \int (\frac{1}{(y-2)}-\frac{1}{y+1})\,dy = \int 3x^2\,dx$ $\Rightarrow \ln(y-2) - \ln(y+1) = x^3 + c$ $\Rightarrow \ln\left(\frac{y-2}{y+1}\right) = x^3 + c$ $\Rightarrow \frac{y-2}{y+1} = e^{x^3+c} = e^{x^3}.e^c = Ae^{x^3}$ *	 M1 B1ft B1 M1 E1 [5]	 separating variables $\ln(y-2) - \ln(y+1)$ ft their A,B $x^3 + c$ anti-logging including c www
6 $\tan(\theta+45) = \frac{\tan\theta+\tan 45}{1-\tan\theta\tan 45}$ $= \frac{\tan\theta+1}{1-\tan\theta}$ $\Rightarrow \frac{\tan\theta+1}{1-\tan\theta} = 1-2\tan\theta$ $\Rightarrow 1+\tan\theta = (1-2\tan\theta)(1-\tan\theta)$ $= 1-3\tan\theta+2\tan^2\theta$ $\Rightarrow 0 = 2\tan^2\theta - 4\tan\theta = 2\tan\theta(\tan\theta-2)$ $\Rightarrow \tan\theta = 0$ or 2 $\Rightarrow \theta = 0$ or 63.43	M1 A1 M1 A1 M1 A1A1 [7]	oe using sin/cos multiplying up and expanding any correct one line equation solving quadratic for tan θ oe www -1 extra solutions in the range

Section B

7(i) $\overrightarrow{AB} = \begin{pmatrix} 100-(-200) \\ 200-100 \\ 100-0 \end{pmatrix} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ * AB = $\sqrt{(300^2 + 100^2 + 100^2)}$ = 332 m	E1 M1 A1 [3]	 accept surds
(ii) $\mathbf{r} = \begin{pmatrix} -200 \\ 100 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ Angle is between $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\Rightarrow \cos\theta = \frac{3\times0+1\times0+1\times1}{\sqrt{11}\sqrt{1}} = \frac{1}{\sqrt{11}}$ $\Rightarrow \theta = 72.45°$	B1B1 M1 M1 A1 A1 [6]	oe …and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ complete scalar product method(including cosine) for correct vectors 72.5º or better, accept 1.26 radians
(iii) Meets plane of layer when $(-200+300\lambda) + 2(100+100\lambda) + 3\times100\lambda = 320$ $\Rightarrow 800\lambda = 320$ $\Rightarrow \lambda = 2/5$ $\mathbf{r} = \begin{pmatrix} -200 \\ 100 \\ 0 \end{pmatrix} + \frac{2}{5}\begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix} = \begin{pmatrix} -80 \\ 140 \\ 40 \end{pmatrix}$ so meets layer at (–80, 140, 40)	 M1 A1 M1 A1 [4]	
(iv) Normal to plane is $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ Angle is between $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\Rightarrow \cos\theta = \frac{3\times1+1\times2+1\times3}{\sqrt{11}\sqrt{14}} = \frac{8}{\sqrt{11}\sqrt{14}} = 0.6446..$ $\Rightarrow \theta = 49.86°$ $\Rightarrow$ angle with layer = 40.1°	B1 M1A1 A1 A1 [5]	 complete method ft 90-theirθ accept radians

8(i) At A, $y = 0 \Rightarrow 4\cos\theta = 0,\ \theta = \pi/2$ At B, $\cos\theta = -1, \Rightarrow \theta = \pi$ x-coord of A $= 2\times\pi/2 - \sin\pi/2 = \pi - 1$ x-coord of B $= 2\times\pi - \sin\pi = 2\pi$ $\Rightarrow$ OA $= \pi - 1$, AC $= 2\pi - \pi + 1 = \pi + 1$ $\Rightarrow$ ratio is $(\pi - 1):(\pi + 1)$ *	B1 B1 M1 A1 E1 [5]	 for either A or B/C for both A and B/C
(ii) $\frac{dy}{d\theta} = -4\sin\theta$ $\frac{dx}{d\theta} = 2 - \cos\theta$ $\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= -\frac{4\sin\theta}{2-\cos\theta}$ At A, gradient $= -\frac{4\sin(\pi/2)}{2-\cos(\pi/2)} = -2$	B1 M1 A1 A1 [4]	either $dx/d\theta$ or $dy/d\theta$ www
(iii) $\frac{dy}{dx} = 1 \Rightarrow -\frac{4\sin\theta}{2-\cos\theta} = 1$ $\Rightarrow -4\sin\theta = 2 - \cos\theta$ $\Rightarrow \cos\theta - 4\sin\theta = 2$ *	M1 E1 [2]	their $dy/dx = 1$
(iv) $\cos\theta - 4\sin\theta = R\cos(\theta + \alpha)$ $= R(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 1, R\sin\alpha = 4$ $\Rightarrow R^2 = 1^2 + 4^2 = 17, R = \sqrt{17}$ $\tan\alpha = 4, \alpha = 1.326$ $\Rightarrow \sqrt{17}\cos(\theta + 1.326) = 2$ $\Rightarrow \cos(\theta + 1.326) = 2/\sqrt{17}$ $\Rightarrow \theta + 1.326 = 1.064, 5.219, 7.348$ $\Rightarrow \theta = (-0.262), 3.89, 6.02$	 M1 B1 M1 A1 M1 A1 A1 [7]	 corr pairs accept 76.0º, 1.33 radians inv cos $(2/\sqrt{17})$ ft their R for method -1 extra solutions in the range