

ADVANCED GCE
MATHEMATICS (MEI)
Methods for Advanced Mathematics (C3)

4753/01

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

- Scientific or graphical calculator

Friday 11 June 2010
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Evaluate $\int_0^{\frac{1}{6}\pi} \cos 3x \, dx$. [3]

2 Given that $f(x) = |x|$ and $g(x) = x + 1$, sketch the graphs of the composite functions $y = fg(x)$ and $y = gf(x)$, indicating clearly which is which. [4]

3 (i) Differentiate $\sqrt{1 + 3x^2}$. [3]

(ii) Hence show that the derivative of $x\sqrt{1 + 3x^2}$ is $\frac{1 + 6x^2}{\sqrt{1 + 3x^2}}$. [4]

4 A piston can slide inside a tube which is closed at one end and encloses a quantity of gas (see Fig. 4).

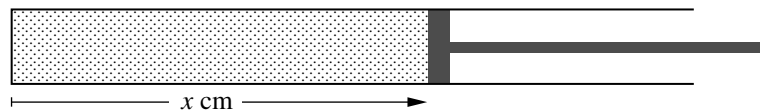


Fig. 4

The pressure of the gas in atmospheric units is given by $p = \frac{100}{x}$, where x cm is the distance of the piston from the closed end. At a certain moment, $x = 50$, and the piston is being pulled away from the closed end at 10 cm per minute. At what rate is the pressure changing at that time? [6]

5 Given that $y^3 = xy - x^2$, show that $\frac{dy}{dx} = \frac{y - 2x}{3y^2 - x}$.

Hence show that the curve $y^3 = xy - x^2$ has a stationary point when $x = \frac{1}{8}$. [7]

6 The function $f(x)$ is defined by

$$f(x) = 1 + 2 \sin 3x, \quad -\frac{\pi}{6} \leq x \leq \frac{\pi}{6}.$$

You are given that this function has an inverse, $f^{-1}(x)$.

Find $f^{-1}(x)$ and its domain. [6]

7 State whether the following statements are true or false; if false, provide a counter-example.

(i) If a is rational and b is rational, then $a + b$ is rational.

(ii) If a is rational and b is irrational, then $a + b$ is irrational.

(iii) If a is irrational and b is irrational, then $a + b$ is irrational. [3]

Section B (36 marks)

- 8 Fig. 8 shows the curve $y = 3 \ln x + x - x^2$.

The curve crosses the x -axis at P and Q, and has a turning point at R. The x -coordinate of Q is approximately 2.05.

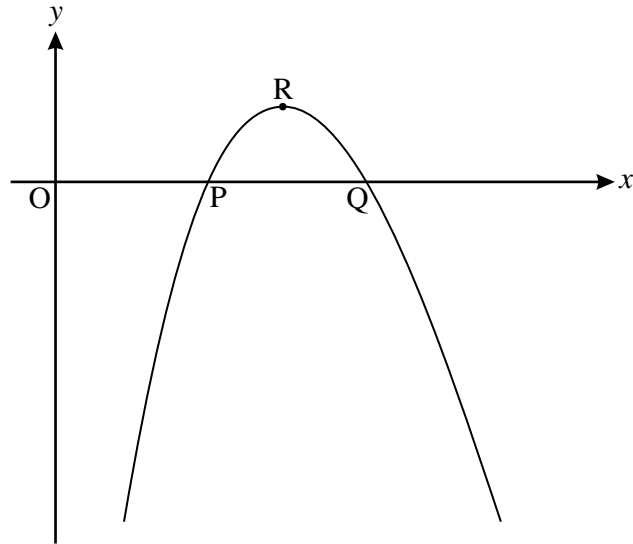


Fig. 8

- (i) Verify that the coordinates of P are (1, 0). [1]

- (ii) Find the coordinates of R, giving the y -coordinate correct to 3 significant figures.

Find $\frac{d^2y}{dx^2}$, and use this to verify that R is a maximum point. [9]

- (iii) Find $\int \ln x \, dx$.

Hence calculate the area of the region enclosed by the curve and the x -axis between P and Q, giving your answer to 2 significant figures. [7]

[Question 9 is printed overleaf.]

- 9 Fig. 9 shows the curve $y = f(x)$, where $f(x) = \frac{e^{2x}}{1 + e^{2x}}$. The curve crosses the y -axis at P.

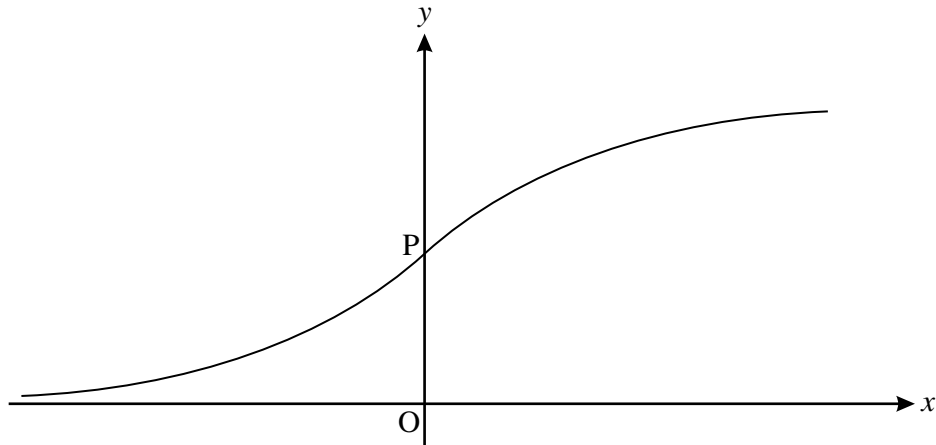


Fig. 9

- (i) Find the coordinates of P. [1]

- (ii) Find $\frac{dy}{dx}$, simplifying your answer.

Hence calculate the gradient of the curve at P. [4]

- (iii) Show that the area of the region enclosed by $y = f(x)$, the x -axis, the y -axis and the line $x = 1$ is $\frac{1}{2} \ln\left(\frac{1 + e^2}{2}\right)$. [5]

The function $g(x)$ is defined by $g(x) = \frac{1}{2} \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right)$.

- (iv) Prove algebraically that $g(x)$ is an odd function.

Interpret this result graphically. [3]

- (v) (A) Show that $g(x) + \frac{1}{2} = f(x)$.

(B) Describe the transformation which maps the curve $y = g(x)$ onto the curve $y = f(x)$.

(C) What can you conclude about the symmetry of the curve $y = f(x)$? [6]

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