

ADVANCED SUBSIDIARY GCE
MATHEMATICS
Further Pure Mathematics 1

4725

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

- Scientific or graphical calculator

Friday 11 June 2010
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

- 1 Prove by induction that, for $n \geq 1$, $\sum_{r=1}^n r(r+1) = \frac{1}{3}n(n+1)(n+2)$. [5]
- 2 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 1 & -4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & 0 \\ -2 & 2 \end{pmatrix}$. Find
- (i) \mathbf{AB} , [2]
- (ii) $\mathbf{BA} - 4\mathbf{C}$. [4]
- 3 Find $\sum_{r=1}^n (2r-1)^2$, expressing your answer in a fully factorised form. [6]
- 4 The complex numbers a and b are given by $a = 7 + 6i$ and $b = 1 - 3i$. Showing clearly how you obtain your answers, find
- (i) $|a - 2b|$ and $\arg(a - 2b)$, [4]
- (ii) $\frac{b}{a}$, giving your answer in the form $x + iy$. [3]
- 5 (a) Write down the matrix that represents a reflection in the line $y = x$. [2]
- (b) Describe fully the geometrical transformation represented by each of the following matrices:
- (i) $\begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$, [2]
- (ii) $\begin{pmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$. [2]
- 6 (i) Sketch on a single Argand diagram the loci given by
- (a) $|z - 3 + 4i| = 5$, [2]
- (b) $|z| = |z - 6|$. [2]
- (ii) Indicate, by shading, the region of the Argand diagram for which
- $$|z - 3 + 4i| \leq 5 \quad \text{and} \quad |z| \geq |z - 6|. \quad [2]$$
- 7 The quadratic equation $x^2 + 2kx + k = 0$, where k is a non-zero constant, has roots α and β . Find a quadratic equation with roots $\frac{\alpha + \beta}{\alpha}$ and $\frac{\alpha + \beta}{\beta}$. [7]

8 (i) Show that $\frac{1}{\sqrt{r+2} + \sqrt{r}} \equiv \frac{\sqrt{r+2} - \sqrt{r}}{2}$. [2]

(ii) Hence find an expression, in terms of n , for

$$\sum_{r=1}^n \frac{1}{\sqrt{r+2} + \sqrt{r}}. \quad [6]$$

(iii) State, giving a brief reason, whether the series $\sum_{r=1}^{\infty} \frac{1}{\sqrt{r+2} + \sqrt{r}}$ converges. [1]

9 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} a & a & -1 \\ 0 & a & 2 \\ 1 & 2 & 1 \end{pmatrix}$.

(i) Find, in terms of a , the determinant of \mathbf{A} . [3]

(ii) Three simultaneous equations are shown below.

$$\begin{aligned} ax + ay - z &= -1 \\ ay + 2z &= 2a \\ x + 2y + z &= 1 \end{aligned}$$

For each of the following values of a , determine whether the equations are consistent or inconsistent. If the equations are consistent, determine whether or not there is a unique solution.

(a) $a = 0$

(b) $a = 1$

(c) $a = 2$

[6]

10 The complex number z , where $0 < \arg z < \frac{1}{2}\pi$, is such that $z^2 = 3 + 4i$.

(i) Use an algebraic method to find z . [5]

(ii) Show that $z^3 = 2 + 11i$. [1]

The complex number w is the root of the equation

$$w^6 - 4w^3 + 125 = 0$$

for which $-\frac{1}{2}\pi < \arg w < 0$.

(iii) Find w . [5]

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