OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4726

Further Pure Mathematics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Thursday 27 May 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

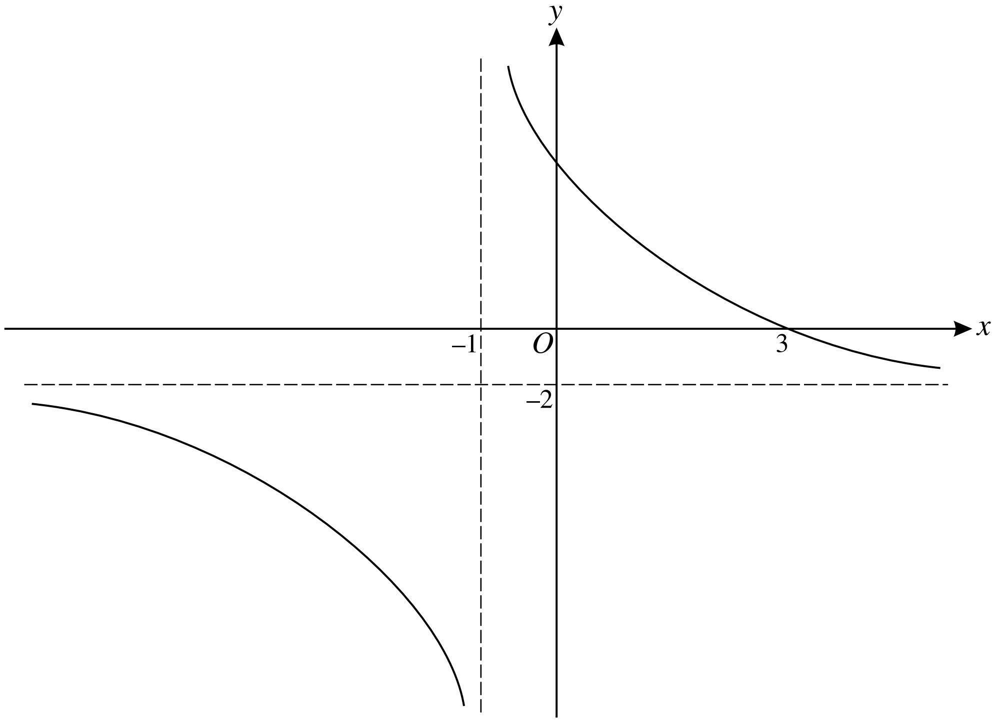
1 It is given that $f(x) = \tan^{-1} 2x$ and $g(x) = p\tan^{-1} x$, where p is a constant. Find the value of p for which $f'\left(\frac{1}{2}\right) = g'\left(\frac{1}{2}\right)$. **[4]**

2 Given that the first three terms of the Maclaurin series for $(1 + \sin x)e^{2x}$ are identical to the first three terms of the binomial series for $(1 + ax)^n$, find the values of the constants a and n. (You may use appropriate results given in the List of Formulae (MF1).) **[6]**

3 Use the substitution $t = \tan\frac{1}{2}x$ to show that

$$\int_0^{\frac{1}{3}\pi} \frac{1}{1 - \sin x}\,dx = 1 + \sqrt{3}.$$ **[6]**

4



The diagram shows the curve with equation

$$y = \frac{ax + b}{x + c},$$

where a, b and c are constants.

(i) Given that the asymptotes of the curve are $x = -1$ and $y = -2$ and that the curve passes through $(3, 0)$, find the values of a, b and c. **[3]**

(ii) Sketch the curve with equation

$$y^2 = \frac{ax + b}{x + c},$$

for the values of a, b and c found in part **(i)**. State the coordinates of any points where the curve crosses the axes, and give the equations of any asymptotes. **[4]**

5 It is given that, for $n \geqslant 0$,

$$I_n = \int_0^{\frac{1}{2}} (1-2x)^n e^x \, dx.$$

(i) Prove that, for $n \geqslant 1$,

$$I_n = 2nI_{n-1} - 1. \quad \textbf{[4]}$$

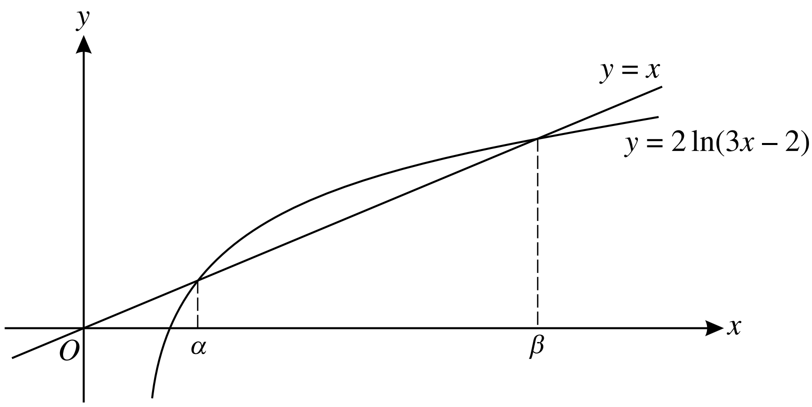
(ii) Find the exact value of I_3. **[4]**

6 **(i)** Show that $\dfrac{d}{dx}(\sinh^{-1} x) = \dfrac{1}{\sqrt{x^2+1}}$. **[2]**

(ii) Given that $y = \cosh(a \sinh^{-1} x)$, where a is a constant, show that

$$(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - a^2 y = 0. \quad \textbf{[5]}$$

7



The line $y = x$ and the curve $y = 2\ln(3x-2)$ meet where $x = \alpha$ and $x = \beta$, as shown in the diagram.

(i) Use the iteration $x_{n+1} = 2\ln(3x_n - 2)$, with initial value $x_1 = 5.25$, to find the value of β correct to 2 decimal places. Show all your working. **[2]**

(ii) With the help of a 'staircase' diagram, explain why this iteration will not converge to α, whatever value of x_1 (other than α) is used. **[3]**

(iii) Show that the equation $x = 2\ln(3x-2)$ can be rewritten as $x = \frac{1}{3}\left(e^{\frac{1}{2}x} + 2\right)$. Use the Newton-Raphson method, with $f(x) = \frac{1}{3}\left(e^{\frac{1}{2}x} + 2\right) - x$ and $x_1 = 1.2$, to find α correct to 2 decimal places. Show all your working. **[4]**

(iv) Given that $x_1 = \ln 36$, explain why the Newton-Raphson method would not converge to a root of $f(x) = 0$. **[2]**

[Questions 8 and 9 are printed overleaf.]

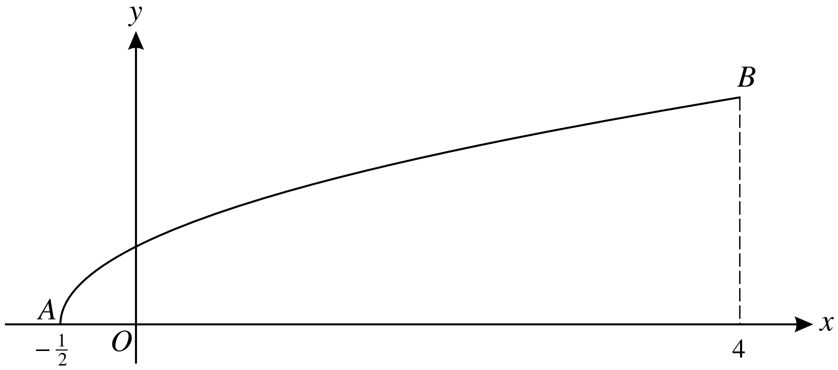
8 **(i)** Using the definition of $\cosh x$ in terms of e^x and e^{-x}, show that

$$4\cosh^3 x - 3\cosh x \equiv \cosh 3x.$$ **[4]**

(ii) Use the substitution $u = \cosh x$ to find, in terms of $5^{\frac{1}{3}}$, the real root of the equation

$$20u^3 - 15u - 13 = 0.$$ **[6]**

9



The diagram shows the curve with equation $y = \sqrt{2x+1}$ between the points $A\,(-\frac{1}{2}, 0)$ and $B\,(4, 3)$.

(i) Find the area of the region bounded by the curve, the x-axis and the line $x = 4$. Hence find the area of the region bounded by the curve and the lines OA and OB, where O is the origin. **[4]**

(ii) Show that the curve between B and A can be expressed in polar coordinates as

$$r = \frac{1}{1 - \cos\theta}, \quad \text{where } \tan^{-1}\left(\tfrac{3}{4}\right) \leqslant \theta \leqslant \pi.$$ **[5]**

(iii) Deduce from parts **(i)** and **(ii)** that $\displaystyle\int_{\tan^{-1}\left(\frac{3}{4}\right)}^{\pi} \operatorname{cosec}^4\left(\tfrac{1}{2}\theta\right)\,d\theta = 24.$ **[4]**