GCE

Mathematics

Advanced GCE

Unit **4734:** Probability and Statistics 3

Mark Scheme for June 2011

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

1 (i)	E(S)= 22 Var(S)=E(S)	B1 B1 **2**	
(ii)	E(T) = ½×5 - ¼×4=1.5 Var(T)=¼×5 + $^1/_{16}$×4 = 1.5 =E(T) AG	B1 M1 A1 **3**	Using Var($aX+bY$) **CWO**
(iii)	T only does not have a Poisson distribution Some values of T are EITHER negative OR: fractional	B1 B1 **2** **(7)**	Unless wrong reason
2(i)	Use $(^6/_{80})(^{74}/_{80})/80$ $p_s \pm zs$ z= 1.96 (0.0173, 0.1327)	B1 M1 B1 A1 **4**	Or /79 s of the form $\sqrt{(p_sq_s/80)}$ (or 79) or no √ Accept (0.017,0.133)
(ii)	Use $z\sqrt{(p_sq_s/n)}$ ≤ 0.05 $n \ge 106.6$, least is 107	M1 A1 A1 **3**	or no √ and z=1.96 .Or = Allow 110
(iii)	e.g Variance is an estimate OR Distribution of p_s is only approx normal	B1 **1** **(8)**	Not var unknown Must state distribution of what.
3(i)	$\int_0^1 ax\,dx + \int_1^2 a(x-2)^2\,dx = 1$ $\left[\frac{ax^2}{2}\right]_0^1 + \left[\frac{a(x-2)^3}{3}\right]_1^2$ ½ a + ⅓a = 1 $a = {}^6/_5$	M1 B1 M1 A1 **4**	With or without limits Correct method for equation with fractions/decimals
(ii)	EITHER: $\int_0^1 ax\,dx + \int_1^{1.5} a(x-2)^2\,dx$ OR $1-\int_{1.5}^2 a(x-2)^2\,dx$ = $^{19}/_{20}$	M1 A1 **2**	Any a AEF
(iii)	$\int_0^1 ax^2\,dx + \int_1^2 ax(x-2)^2\,dx$ $= \left[\frac{ax^3}{3}\right]_0^1 + \left[a(\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2\right]_1^2$ =9/10 (Expected monthly demand = 900)	M1 B1 A1 **3** **(9)**	AEF With or without limits AEF

4(i)	$2608p$ $p = e^{-3.87}3.87^6/6!$ (×2608=253.82) $(273-253.82)^2/253.82$ =1.449	M1 A1 M1 A1 **4**	p from Poisson From 253.8 or 254 seen Answer between 1.445 and 1.460
(ii)	Number of cells – 1 (estimated mean) – 1(same totals)	B1 **1**	Not 11-1
(iii)	H_0: A Poisson distribution fits the data H_1: A Poisson distribution does not fit the data CV = 15. 99 13.0 < CV and do not reject Ho accept that there is insufficient evidence that a Poisson distribution does not fit data	 B1 B1 M1 A1 **4** **(9)**	For both hypotheses Their CV Sufficient evidence that Poisson distribution fits data , OK
5(i)	Solve $\frac{4}{3}(1-\frac{1}{m^2}) = \frac{1}{2}$ Giving $m = \sqrt{\frac{8}{5}}$	M1 A1 **2**	 Or equivalent. 1.26, 1.265, 2√10/5
(ii)	$G(y) = P(Y \le y)$ or < $= P(X \ge 1/\sqrt{y})$ $= 1 - F(1/\sqrt{y})$ $= 1 - {}^4/_3(1-y)$ or $(4y-1)/3$ $1 \le 1/\sqrt{y} \le 2 \Rightarrow ¼ \le y \le 1$ $g(y) = \begin{cases} 4/3 & 1/4 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$	M1 A1 M1 A1 B1 B1√ **6**	Or:$x=1/\sqrt{y}$, $\lvert dx/dy\rvert=1/(2y^{3/2})$ B1 $f(x)=8/(3x^3)$; $1\le x\le 2$ M1A1 $g(y)=f(x)\lvert dx/dy\rvert$ M1 =4/3 A1 $¼\le y\le 1$ B1 Ft G(y)
(iii)	EITHER: E(2-2Y) $= 2 - 2\times{}^5/_8$ $= {}^3/_4$ OR $2 - \int_1^2 16/(3x^5)dx$ OR $\int_1^2 (2-2/x^2))(8/3x^3)dx$ $= 2 + [4/(3x^4)]$ $=[-8/(3x^2) +4/(3x^4)]$ = 3/4 =3/4	M1 A1√ A1 M1 A1 A1 **3** **(11)**	 √ g(y) CAO AEF From 2 - $\int xF'(x)dx$ √ f(x) CAO AEF
6(i)	$s^2 = (68636.41 - 2605^2/100)/99$ (=7.84) $\bar{x} = 26.05$ $26.05 \pm zs/10$ $z = 2.326$ or $\Phi^{-1}(0.99)$ ART (25.4, 26.7)	B1 B1 M1 B1 A1 **5**	AEF Allow t(99)=2.365
(ii)	Use N(26.05,7.84) $P(\ge 30)= 1-\Phi([30-26.05]/\sqrt{7.84})$ = 0.0792 = 7.92%	M1 M1 A1 **3**	s^2 from (i) M0 for 7.84/100 No “cc” allow either; ART 0.08 or 8%
(iii)	Use $B_1 - B_2 \sim N(0, 15.68)$ $P(< 5) = \Phi(5/\sigma)$ = 0.897	M1 A1 A1 A1 **4**	With $\mu = 0$ For variance σ^2 Their σ; $\Phi(\pm 5/\sigma) \Rightarrow$ M1
(iv)	(i) only since sample size of 100 is large enough (for CLT to hold)	B1 **1** **(13)**	Must be clear which part and with correct reason.

7(i)	For each student the scores are correlated	B1 **1**	Or equivalent**,** eg paired
(ii)	Increase in score has a normal distribution Sample is considered to be a random sample of all students attending the course H_0: μ_D = 0, H_1: μ_D > 0 where *D*= increase in scores D= 10 2 12 -3 18 10 11 6 14 9 $\bar{D}$ = 8.9 s^2 = 35.88 Test statistic = 8.9/(*s*/√10) = 4.699 *v* = 9, CV = 3.25 4.699 > CV Reject H_0 and accept that there is sufficient evidence a at the ½ % significance level of an increase in mean scores. SR 2-sample test: (i)B0(ii)B0B1B1M0 Max 2/11	B1 B1 B1 M1 B1 B1 M1 A1 B1 M1 **10**	Allow pop of differences~ normal Or equivalent , allow independent Or H_0: μ_1=μ_2 H_1: μ_1 < μ_2 : not μ =0 D may be implied Must involve 10 Allow ART 4.70 Or P(*t*>4.699)=0.00056<0.005 Not OA
(iii)	Test statistic = (8.9-5)/√3.588=2.059 This is significant of an increase at the 5% significance level (CV of 1.833) so director's claim is supported. SR 2-sample t-test. (8.9-5)/s M1 Max1/4 SR: Use of confidence intervals 99% C I 2-sided (2.74,15.1) : 99.5% 1-sided (2.74, ∞) 5 is in this interval so not significant at ½ % level A1 OR 90% CI 2-sided (5.43,12.37) ; 95% 1-sided (5.43, ∞) 5 not in this interval so significant at 5% SL	M1A1 M1 A1 **4** **(15)** M1A1 M1 A1 M1A1 M1 A1	Or P(*t*>2.059)=0.035 Any reasonable significance level with corresponding conclusion Allow at ½ %

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553