RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4735:** Probability and Statistics 4

Mark Scheme for June 2011

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1 (i)	$\sum_{x=0}^{n}\binom{n}{x}p^x q^{n-x}t^x$ $=\sum_{x=0}^{n}\binom{n}{x}(pt)^x q^{n-x}$	M1 A1 **2**	From E(t^x) M1A0 ∑ without limits $G_X(t)$=q+pt M1 then argument A0
(ii)	$G_T(t) = (q+pt)^n(q+pt)^{2n}$ $= (q+pt)^{3n}$ So $T \sim B(3n, p)$	M1 A1 M1 A1 **4** **[6]**	Multiplying pgfs For B For parameters
2 (i)	H_0: $m_d = 0$, H_1: $m_d > 0$, (where d = high – low) D: -4 3 6 1 12 7 14 16 11 -9 10 Rank -3 2 4 1 9 5 10 11 8 -6 7 $P = 57$, $Q = 9$ $T = 9$ CV = 13 9 < CV so reject H_0 There is sufficient evidence at the 5% significance level to support the botanist's belief	B1 M1 A1 B1 B1 M1 A1 ft **7**	Or H_o:m_H=m_L , etc. Medians Ranking top down, -9,-10,8, ..M1A0 T=15 B0 [SR last 3 marks: z=-2;09 B1 <-1.96 etc M1A1] Or equivalent ft T
(ii)	The rank sum test is for independent samples, the H and L values are correlated	B1 **1** **[8]**	Accept data paired
3 (i)	$P(A\|B') = P(A\cap B') / P(B')$ => $P(A\cap B')$ = 1/8 AEF Use $P(A\cap B) = P(A) - P(A\cap B')$ To give $P(A\cap B)$ = 5/8 AEF	M1 A1 M1 A1 **4**	 May be implied Or equivalent
(ii)	$P(A\cap B\cap C) = {}^5/_8\times{}^1/_4 = {}^5/_{32}$ AEF	B1 √ **1**	Ft 5/8
(iii)	$P(B\cap C) = 3\lambda/4$ and $P(C\cap A) = 3\lambda/4$ Use formula for $P(A\cup B\cup C)$ And $P(A\cup B\cup C)$=1 Sub into formula for $P(A\cup B\cup C)$ and solve for λ giving λ = 3/16 AEF	M1 M1 B1 M1 A1 **5** **[10]**	For use of both conditional probs Allow one sign error
4 (i)	$M'(t) = 3({}^1/_4 + {}^3/_4e^t)^2\times{}^3/_4\, e^t$ $E(X) = M'(0) = {}^9/_4$	M1 A1 A1 **3**	Allow one error
(ii)	mgf $({}^1/_{64} + {}^9/_{64}e^t +)$ ${}^{27}/_{64}e^{2t}$ $(+\ {}^{27}/_{64}e^{3t})$ $P(X = 2)$ = coefficient of $e^{2t} = {}^{27}/_{64}$	M1 A1 A1 **3**	Or PGF=$({}^1/_4 + {}^3/_4 z)^3$ expand find coefficient of z^2 27/64
(iii)	Sum of 3 obs of Y with mgf ¼ + ¾e^t has mgf of X y : 0 1 p: ¼ ¾ $Var(Y)$ = ¾ - (¾)^2 = ${}^3/_{16}$	M1*dep A1 *M1A1 **4** **[10]**	 OR B(1,${}^3/_4$) Using $E(Y^2) - (E(Y))^2$ OR $1\times{}^3/_4\times{}^1/_4$ M0 if integration used

5(i)	Does not require a known probability distribution	B1 **1**	Or equivalent
(ii)	H_0: $m_A = m_B$, H_1: $m_A \neq m_B$ Ranks: A 1 2 3 5 6 10 B 4 7 8 9 11 12 R_A = 27, 78 – 27 = 51, so W = 27 OR: R_B = 51, 78 – 51 = 27 5% CV = 26 27 > CV so do not reject H_0 there is insufficient evidence at the 5% SL to indicate a difference in breaking strengths	B1 M1 M1 A1 B1 M1 A1 **7**	Medians Use N(39,39) with cc B1 P(W≤27.5), Z=-1.84 or equivalent M1 Not in CR etc A1
(iii)	B would have an extra rank 13 W still 27 but CV now 27 H_0 is now rejected	M1 B1 A1 **3** **[11]**	 P(W≤27.5)=-2.07 M1A1 In CR Ho rejected A1
6(i)	L=0, C = 1, choose 1C from 14 and 1 from 6 Others $14\times6 / {}^{36}C_2 = 2/15$ AG L = 1, C = 1, choose 1 from 16, 1 from 14 $16\times14 / {}^{36}C_2 = 16/45$ AG	M1 A1 M1 A1 **4**	Or $^{14}/_{36}\times{}^{6}/_{35}\times2$ Or $^{14}/_{36}\times{}^{16}/_{35}\times2$
(ii)	Marginal C probs: 11/30 22/45 13/90 E(C) = 22/45 + 26/90 = 35/45 = 7/9	B1 M1 A1 **3**	 AEF
(iii)	EITHER: 2×1/42 + 2/15 + 16/105 OR: E(L) = 8/9, E(O) = 2 – 15/9 = 1/3	M1 A1 A1 **3**	Other: 0 1 2 M1 p: $^{29}/_{42}$ $^{2}/_{7}$ $^{1}/_{42}$ A1 E(O)=$^{2}/_{7}$ + $^{2}/_{42}$=$^{1}/_{3}$ A1
(iv)	EITHER: Argument OR: Use idea that for independence $P(L \cap C) = P(L)P(C)$ Conclude that covariance is non-zero	B2 M1A1 B1 **3** **[13]**	e.g The more Ls the fewer Cs OR Use conditional probability OR Cov(L,C)=-136/405 M1A1 L,C not indep B1
7(i)	$E(S) = \frac{1}{2}(E(\bar{U}_4) + E(\bar{U}_6))$ $= \frac{1}{2}(\mu + \mu) = \mu$, so S is unbiased $Var(S) = \frac{1}{4}(\sigma^2/4 + \sigma^2/6)$ $= 5\sigma^2/48$	M1 A1 M1 A1 **4**	 With conclusion
(ii)	$E(T) = (a+b)\mu = \mu$, $a+b=1$ $Var(T) = a^2\sigma^2/4 + b^2\sigma^2/6$ Minimise $y = a^2/4 + b^2/6 = a^2/4 + (1-a)^2/6$ EITHER by differentiation OR, completing square, OR from a sketch graph. Giving a = 2/5, b = 3/5 Justify minimum value Variance = $\sigma^2/10$	M1 B1 M1 M1 A1 B1 A1 **7**	 Allow from completion of square
(iii)	T is better since (both are unbiased and) Var(T)<Var(S)	B1 **1**	From calculated variances
(iv)	Sample mean of 10 observations (is also unbiased) with $\sigma^2/10$ They have the same efficiency	M1 A1 **2** **[14]**	Or show that T = mean of 10 observations

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