

ADVANCED GCE MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:

- Printed answer book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

• Scientific or graphical calculator

Monday 20 June 2011 Morning

4753/01

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the printed answer book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [] at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

• Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

- Given that $f(x) = 2 \ln x$ and $g(x) = e^x$, find the composite function gf(x), expressing your answer as 2 simply as possible. [3]
- (i) Differentiate $\frac{\ln x}{x^2}$, simplifying your answer. 3 [4]

(ii) Using integration by parts, show that
$$\int \frac{\ln x}{x^2} \, dx = -\frac{1}{x}(1+\ln x) + c.$$
 [4]

4 The height h metres of a tree after t years is modelled by the equation

$$h = a - b \mathrm{e}^{-kt},$$

where *a*, *b* and *k* are positive constants.

- (i) Given that the long-term height of the tree is 10.5 metres, and the initial height is 0.5 metres, find the values of *a* and *b*. [3]
- (ii) Given also that the tree grows to a height of 6 metres in 8 years, find the value of k, giving your answer correct to 2 decimal places. [3]

5 Given that
$$y = x^2 \sqrt{1+4x}$$
, show that $\frac{dy}{dx} = \frac{2x(5x+1)}{\sqrt{1+4x}}$. [5]

- A curve is defined by the equation $\sin 2x + \cos y = \sqrt{3}$. 6
 - (i) Verify that the point $P(\frac{1}{6}\pi, \frac{1}{6}\pi)$ lies on the curve. [1]
 - (ii) Find $\frac{dy}{dx}$ in terms of x and y.

Hence find the gradient of the curve at the point P. [5]

7 (i) Multiply out
$$(3^n + 1)(3^n - 1)$$
. [1]

(ii) Hence prove that if n is a positive integer then $3^{2n} - 1$ is divisible by 8. [3]

Section B (36 marks)



Fig. 8 shows the curve y = f(x), where $f(x) = \frac{1}{e^x + e^{-x} + 2}$.

(i) Show algebraically that f(x) is an even function, and state how this property relates to the curve y = f(x). [3]

(ii) Find
$$f'(x)$$
. [3]

(iii) Show that
$$f(x) = \frac{e^x}{(e^x + 1)^2}$$
. [2]

- (iv) Hence, using the substitution $u = e^x + 1$, or otherwise, find the exact area enclosed by the curve y = f(x), the *x*-axis, and the lines x = 0 and x = 1. [5]
- (v) Show that there is only one point of intersection of the curves y = f(x) and $y = \frac{1}{4}e^x$, and find its coordinates. [5]

[Question 9 is printed overleaf.]

8

9 Fig. 9 shows the curve y = f(x). The endpoints of the curve are $P(-\pi, 1)$ and $Q(\pi, 3)$, and $f(x) = a + \sin bx$, where a and b are constants.





- (i) Using Fig. 9, show that a = 2 and $b = \frac{1}{2}$. [3]
- (ii) Find the gradient of the curve y = f(x) at the point (0, 2).

Show that there is no point on the curve at which the gradient is greater than this. [5]

(iii) Find $f^{-1}(x)$, and state its domain and range.

Write down the gradient of
$$y = f^{-1}(x)$$
 at the point (2, 0). [6]

(iv) Find the area enclosed by the curve y = f(x), the x-axis, the y-axis and the line $x = \pi$. [4]



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