OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4754A

Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4754A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Monday 13 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES
These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES
This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 Express $\dfrac{1}{(2x+1)(x^2+1)}$ in partial fractions. **[5]**

2 Find the first three terms in the binomial expansion of $\sqrt[3]{1+3x}$ in ascending powers of x. State the set of values of x for which the expansion is valid. **[5]**

3 Express $2\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants to be determined, and $0 < \alpha < \frac{1}{2}\pi$.

Hence write down the greatest and least possible values of $1 + 2\sin\theta - 3\cos\theta$. **[6]**

4 A curve has parametric equations

$$x = 2\sin\theta, \quad y = \cos 2\theta.$$

(i) Find the exact coordinates and the gradient of the curve at the point with parameter $\theta = \frac{1}{3}\pi$. **[5]**

(ii) Find y in terms of x. **[2]**

5 Solve the equation $\operatorname{cosec}^2\theta = 1 + 2\cot\theta$, for $-180° \leqslant \theta \leqslant 180°$. **[6]**

6 Fig. 6 shows the region enclosed by part of the curve $y = 2x^2$, the straight line $x + y = 3$, and the y-axis. The curve and the straight line meet at P (1, 2).

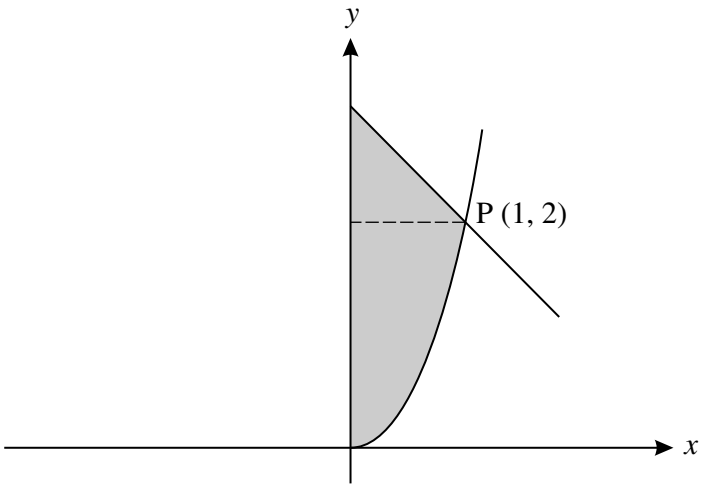

Fig. 6

The shaded region is rotated through 360° about the y-axis. Find, in terms of π, the volume of the solid of revolution formed. **[7]**

[You may use the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone.]

Section B (36 marks)

7 A piece of cloth ABDC is attached to the tops of vertical poles AE, BF, DG and CH, where E, F, G and H are at ground level (see Fig. 7). Coordinates are as shown, with lengths in metres. The length of pole DG is k metres.

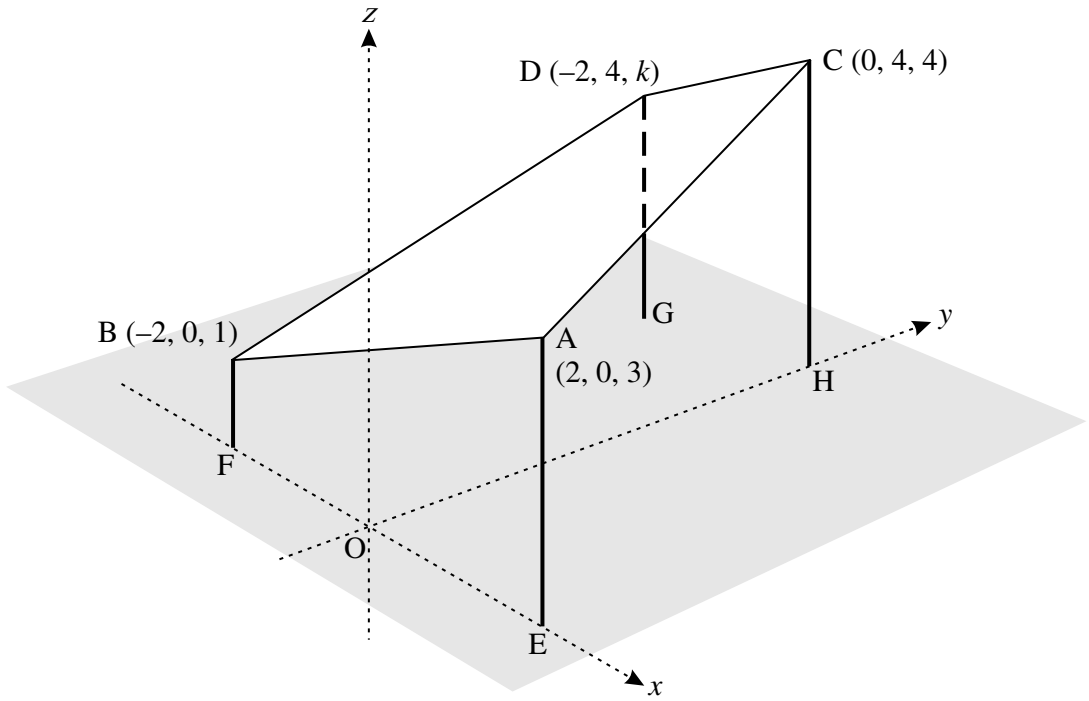

Fig. 7

(i) Write down the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Hence calculate the angle BAC. **[6]**

(ii) Verify that the equation of the plane ABC is $x + y - 2z + d = 0$, where d is a constant to be determined.

Calculate the acute angle the plane makes with the horizontal plane. **[7]**

(iii) Given that A, B, D and C are coplanar, show that $k = 3$.

Hence show that ABDC is a trapezium, and find the ratio of CD to AB. **[5]**

[Question 8 is printed overleaf.]

8 Water is leaking from a container. After t seconds, the depth of water in the container is x cm, and the volume of water is V cm^3, where $V = \frac{1}{3}x^3$. The rate at which water is lost is proportional to x, so that $\dfrac{\mathrm{d}V}{\mathrm{d}t} = -kx$, where k is a constant.

(i) Show that $x\dfrac{\mathrm{d}x}{\mathrm{d}t} = -k$. **[3]**

Initially, the depth of water in the container is 10 cm.

(ii) Show by integration that $x = \sqrt{100 - 2kt}$. **[4]**

(iii) Given that the container empties after 50 seconds, find k. **[2]**

Once the container is empty, water is poured into it at a constant rate of 1 cm^3 per second. The container continues to lose water as before.

(iv) Show that, t seconds after starting to pour the water in, $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{1-x}{x^2}$. **[2]**

(v) Show that $\dfrac{1}{1-x} - x - 1 = \dfrac{x^2}{1-x}$.

Hence solve the differential equation in part **(iv)** to show that

$$t = \ln\left(\frac{1}{1-x}\right) - \tfrac{1}{2}x^2 - x.$$ **[6]**

(vi) Show that the depth cannot reach 1 cm. **[1]**

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ADVANCED GCE

MATHEMATICS (MEI) 4754B

Applications of Advanced Mathematics (C4) Paper B: Comprehension

Candidates answer on the question paper.

OCR supplied materials:
- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Rough paper

Monday 13 June 2011
Morning

Duration: Up to 1 hour

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES

- The insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- The insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **4** pages. Any blank pages are indicated.

1 In lines 59 and 60, the text says "In that case the proportion suffering such an attack would be 6.4%."

Explain how this figure was obtained. **[1]**

1	

2 **(i)** In lines 8 to 10, the article says "Some banks do not allow numbers that begin with zero, numbers in which the digits are all the same (such as 5555) or numbers in which the digits are consecutive (such as 2345 or 8765)."

How many different 4-digit PINs can be made when all these rules are applied? **[3]**

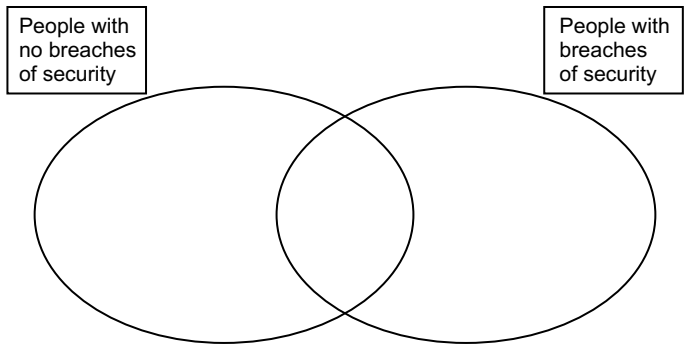
(ii) At the time of writing, the world population is 6.7×10^9 people. Assuming that, on average, each person has one card with a 4-digit PIN (subject to the rules in part **(i)** of this question), estimate the average number of people holding cards with any given PIN. Give your answer to an appropriate degree of accuracy. **[2]**

2 (i)	
2 (ii)	

3 In lines 46 and 47, the text says "Of the 11 people with unauthorised transactions, 3 could explain them as breaches of card security (typically losing the card) but 9 could not … ."

Place numbers in the three regions of the diagram consistent with the information in this sentence. **[2]**

3



4 In lines 101 and 102, the text says "The total number of transactions for those who responded has been estimated as 100 000 for the $3\frac{1}{2}$ years covered by the survey."

Estimate the number of transactions per person per day that would give this figure. **[2]**

4

5 The survey described in the article was based on a small sample.

State one conclusion which is unlikely to be influenced by the size of the sample. **[1]**

5

6 A bank has detection software that can be set at two different levels, 'Mild' and 'Severe'.

- When it is set at Mild, 0.1% of all transactions are queried.
- When it is set at Severe 0.5% of all transactions are queried.

(i) One day the bank has 500 000 transactions.

The software is set on 'Mild'. There are 480 false positives. Only $\frac{1}{3}$ of the unauthorised transactions are queried. Complete the table. **[3]**

(ii) What is the ratio of false positives to false negatives? **[1]**

(iii) If the software had been set on 'Severe' for the same set of 500 000 transactions, with the total numbers of authorised and unauthorised transactions the same as in part **(i)** of this question, the number of false negatives would have been 5. What would the ratio of false positives to false negatives have been with this setting? **[3]**

6 (i)

Transactions	Authorised	Unauthorised	Total
Queried			
Not queried			
Total			500 000

6 (ii)

6 (iii)

[A copy of the table is provided below for your working.]

Transactions	Authorised	Unauthorised	Total
Queried			
Not queried		5	
Total			500 000