

**ADVANCED SUBSIDIARY GCE  
MATHEMATICS**

Further Pure Mathematics 1

**4725**

**QUESTION PAPER**

Candidates answer on the printed answer book.

**OCR supplied materials:**

- Printed answer book 4725
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Thursday 16 June 2011  
Afternoon**

**Duration:** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER / INVIGILATOR**

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

- 1 The matrices **A** and **B** are given by  $\mathbf{A} = \begin{pmatrix} 2 & a \\ 0 & 1 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 2 & a \\ 4 & 1 \end{pmatrix}$ . **I** denotes the  $2 \times 2$  identity matrix. Find

(i)  $\mathbf{A} + 3\mathbf{B} - 4\mathbf{I}$ , [3]

(ii)  $\mathbf{AB}$ . [2]

- 2 Prove by induction that, for  $n \geq 1$ ,  $\sum_{r=1}^n \frac{1}{r(r+1)} = \frac{n}{n+1}$ . [5]

- 3 By using the determinant of an appropriate matrix, find the values of  $k$  for which the simultaneous equations

$$kx + 8y = 1,$$

$$2x + ky = 3,$$

do not have a unique solution. [3]

- 4 Find  $\sum_{r=1}^{2n} (3r^2 - \frac{1}{2})$ , expressing your answer in a fully factorised form. [6]

- 5 The complex number  $1 + i\sqrt{3}$  is denoted by  $a$ .

(i) Find  $|a|$  and  $\arg a$ . [2]

(ii) Sketch on a single Argand diagram the loci given by  $|z - a| = |a|$  and  $\arg(z - a) = \frac{1}{2}\pi$ . [6]

- 6 The matrix **C** is given by  $\mathbf{C} = \begin{pmatrix} a & 1 & 0 \\ 1 & 2 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ , where  $a \neq 1$ . Find  $\mathbf{C}^{-1}$ . [7]

- 7 (i) Show that  $\frac{1}{r-1} - \frac{1}{r+1} \equiv \frac{2}{r^2-1}$ . [1]

(ii) Hence find an expression, in terms of  $n$ , for  $\sum_{r=2}^n \frac{2}{r^2-1}$ . [5]

(iii) Find the value of  $\sum_{r=1000}^{\infty} \frac{2}{r^2-1}$ . [3]

8 The matrix  $\mathbf{X}$  is given by  $\mathbf{X} = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}$ .

- (i) The diagram in the printed answer book shows the unit square  $OABC$ . The image of the unit square under the transformation represented by  $\mathbf{X}$  is  $OA'B'C'$ . Draw and label  $OA'B'C'$ . [3]
- (ii) The transformation represented by  $\mathbf{X}$  is equivalent to a transformation A, followed by a transformation B. Give geometrical descriptions of possible transformations A and B and state the matrices that represent them. [4]

9 One root of the quadratic equation  $x^2 + ax + b = 0$ , where  $a$  and  $b$  are real, is  $16 - 30i$ .

- (i) Write down the other root of the quadratic equation. [1]
- (ii) Find the values of  $a$  and  $b$ . [4]
- (iii) Use an algebraic method to solve the quartic equation  $y^4 + ay^2 + b = 0$ . [7]

10 The cubic equation  $x^3 + 3x^2 + 2 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .

- (i) Use the substitution  $x = \frac{1}{\sqrt{u}}$  to show that  $4u^3 + 12u^2 + 9u - 1 = 0$ . [5]

- (ii) Hence find the values of  $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$  and  $\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2}$ . [5]

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