OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4725

Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4725
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Thursday 16 June 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & a \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & a \\ 4 & 1 \end{pmatrix}$. **I** denotes the 2×2 identity matrix. Find

(i) $\mathbf{A} + 3\mathbf{B} - 4\mathbf{I}$, **[3]**

(ii) $\mathbf{AB}$. **[2]**

2 Prove by induction that, for $n \geqslant 1$, $\displaystyle\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$. **[5]**

3 By using the determinant of an appropriate matrix, find the values of k for which the simultaneous equations

$$kx + 8y = 1,$$
$$2x + ky = 3,$$

do not have a unique solution. **[3]**

4 Find $\displaystyle\sum_{r=1}^{2n} \left(3r^2 - \tfrac{1}{2}\right)$, expressing your answer in a fully factorised form. **[6]**

5 The complex number $1 + i\sqrt{3}$ is denoted by a.

(i) Find $|a|$ and $\arg a$. **[2]**

(ii) Sketch on a single Argand diagram the loci given by $|z - a| = |a|$ and $\arg(z - a) = \frac{1}{2}\pi$. **[6]**

6 The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} a & 1 & 0 \\ 1 & 2 & 1 \\ -1 & 3 & 4 \end{pmatrix}$, where $a \neq 1$. Find $\mathbf{C}^{-1}$. **[7]**

7 **(i)** Show that $\dfrac{1}{r-1} - \dfrac{1}{r+1} \equiv \dfrac{2}{r^2 - 1}$. **[1]**

(ii) Hence find an expression, in terms of n, for $\displaystyle\sum_{r=2}^{n} \frac{2}{r^2 - 1}$. **[5]**

(iii) Find the value of $\displaystyle\sum_{r=1000}^{\infty} \frac{2}{r^2 - 1}$. **[3]**

8 The matrix $\mathbf{X}$ is given by $\mathbf{X} = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}$.

(i) The diagram in the printed answer book shows the unit square $OABC$. The image of the unit square under the transformation represented by $\mathbf{X}$ is $OA'B'C'$. Draw and label $OA'B'C'$. **[3]**

(ii) The transformation represented by $\mathbf{X}$ is equivalent to a transformation A, followed by a transformation B. Give geometrical descriptions of possible transformations A and B and state the matrices that represent them. **[4]**

9 One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is $16 - 30\mathrm{i}$.

(i) Write down the other root of the quadratic equation. **[1]**

(ii) Find the values of a and b. **[4]**

(iii) Use an algebraic method to solve the quartic equation $y^4 + ay^2 + b = 0$. **[7]**

10 The cubic equation $x^3 + 3x^2 + 2 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \dfrac{1}{\sqrt{u}}$ to show that $4u^3 + 12u^2 + 9u - 1 = 0$. **[5]**

(ii) Hence find the values of $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} + \dfrac{1}{\gamma^2}$ and $\dfrac{1}{\alpha^2\beta^2} + \dfrac{1}{\beta^2\gamma^2} + \dfrac{1}{\gamma^2\alpha^2}$. **[5]**