OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4733

Probability & Statistics 2

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4733
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Wednesday 22 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 In Fisher Avenue there are 263 houses, numbered 1 to 263. Explain how to obtain a random sample of 20 of these houses. **[3]**

2 The random variable Y has the distribution $N(\mu, \sigma^2)$. It is given that

$$P(Y < 48.0) = P(Y > 57.0) = 0.0668.$$

Find the value y_0 such that $P(Y > y_0) = 0.05$. **[7]**

3 The random variable X has the distribution $N(\mu, 5^2)$. A hypothesis test is carried out of H_0: $\mu = 20.0$ against H_1: $\mu < 20.0$, at the 1% level of significance, based on the mean of a sample of size 16. Given that in fact $\mu = 15.0$, find the probability that the test results in a Type II error. **[7]**

4 A continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{3}{16}(x-2)^2 & 0 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Sketch the graph of $y = f(x)$. **[2]**

(ii) Calculate the variance of X. **[5]**

(iii) A student writes "X is more likely to occur when x takes values further away from 2". Explain whether you agree with this statement. **[1]**

5 A travel company finds from its records that 40% of its customers book with travel agents. The company redesigns its website, and then carries out a survey of 10 randomly chosen customers. The result of the survey is that 1 of these customers booked with a travel agent.

(i) Test at the 5% significance level whether the percentage of customers who book with travel agents has decreased. **[7]**

(ii) The managing director says that "Our redesigned website has resulted in a decrease in the percentage of our customers who book with travel agents." Comment on this statement. **[1]**

6 Records show that before the year 1990 the maximum daily temperature $T\,^\circ$C at a seaside resort in August can be modelled by a distribution with mean 24.3. The maximum temperatures of a random sample of 50 August days since 1990 can be summarised by

$$n = 50, \qquad \Sigma t = 1314.0, \qquad \Sigma t^2 = 36\,602.17.$$

(i) Test, at the 1% significance level, whether there is evidence of a change in the mean maximum daily temperature in August since 1990. **[11]**

(ii) Give a reason why it is possible to use the Central Limit Theorem in your test. **[1]**

7 The number of customer complaints received by a company per day is denoted by X. Assume that X has the distribution Po(2.2).

(i) In a week of 5 working days, the probability there are at least n customer complaints is 0.146 correct to 3 significant figures. Use tables to find the value of n. **[3]**

(ii) Use a suitable approximation to find the probability that in a period of 20 working days there are fewer than 38 customer complaints. **[5]**

A week of 5 working days in which at least n customer complaints are received, where n is the value found in part **(i)**, is called a 'bad' week.

(iii) Use a suitable approximation to find the probability that, in 40 randomly chosen weeks, more than 7 are bad. **[6]**

8 **(a)** A group of students is discussing the conditions that are needed if a Poisson distribution is to be a good model for the number of telephone calls received by a fire brigade on a working day.

(i) Alice says "Events must be independent". Explain why this condition may not hold in this context. **[1]**

(ii) State a different condition that is needed. Explain whether it is likely to hold in this context. **[2]**

(b) The random variables R, S and T have independent Poisson distributions with means λ, μ and $\lambda + \mu$ respectively.

(i) In the case $\lambda = 2.74$, find $\mathrm{P}(R > 2)$. **[3]**

(ii) In the case $\lambda = 2$ and $\mu = 3$, find $\mathrm{P}(R = 0 \text{ and } S = 1) + \mathrm{P}(R = 1 \text{ and } S = 0)$. Give your answer correct to 4 decimal places. **[3]**

(iii) In the general case, show algebraically that

$$\mathrm{P}(R = 0 \text{ and } S = 1) + \mathrm{P}(R = 1 \text{ and } S = 0) = \mathrm{P}(T = 1). \quad \textbf{[4]}$$