RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4754A:** Applications of Advanced Mathematics: Paper A

Mark Scheme for June 2012

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he / she wishes to be marked, then examiners should do as the candidate requests.
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1			$\frac{4x}{x+1} - \frac{3}{2x+1} = 1$		
			$\Rightarrow \quad 4x(2x+1) - 3(x+1) = (x+1)(2x+1)$	M1	Multiplying throughout by $(2x + 1)(x + 1)$ or combining fractions and multiplying up oe (eg can retain denominator throughout) Condone a single numerical error, sign error or slip provided that there is no conceptual error in the process involved Do not condone omission of brackets unless it is clear from subsequent work that they were assumed eg $4x(2x + 1) - 3(x + 1) = (x + 1)(2x \underline{- 1})$ gets M1 $4x(2x + 1) - 3(x + 1) = 1$ gets M0 $4x(x + 1)(2x + 1) - 3(x + 1)(2x + 1) = (x + 1)(2x + 1)$ gets M0 $4x(2x + 1) - 3(x + 1) = (x + 1)$ gets M1, just, for slip in omission of $(2x + 1)$
			$\Rightarrow \quad 8x^2 + 4x - 3x - 3 = 2x^2 + 3x + 1$ $\Rightarrow \quad 6x^2 - 2x - 4 = 0$	DM1	Multiplying out, collecting like terms and forming quadratic = 0. Follow through from their equation provided the algebra is not significantly eased and it is a quadratic. Condone a further sign or numerical error or minor slip when rearranging.
			$\Rightarrow \quad 3x^2 - x - 2 = 0$	A1	or $6x^2 - 2x - 4 = 0$ oe www, (not fortuitously obtained - check for double errors)
			$\Rightarrow \quad (3x + 2)(x - 1) = 0$	M1	Solving their three term quadratic provided $b^2 - 4ac \geq 0$. Use of <u>correct</u> quadratic equation formula (can be an error when substituting into correct formula) or factorising (giving their correct x^2 and constant terms when factors multiplied out) or comp the square oe. soi
			$\Rightarrow \quad x = -2/3$ or 1	A1	cao for both obtained www (accept $-4/6$ oe, or exact decimal equivalent (condone -0.667or better))
				[5]	SC B1 $x = 1$ with or without any working

Question			Answer	Marks	Guidance
2			$(1+2x)^{1/2} = 1 + \frac{1}{2}(2x) + \frac{\frac{1}{2}.(-\frac{1}{2})}{2!}(2x)^2 + \frac{\frac{1}{2}.(-\frac{1}{2})(-\frac{3}{2})}{3!}(2x)^3 + ...$	M1	Do not MR for $n \neq 1/2$ All four correct binomial coeffs (not nCr form) soi Accept **unsimplified** coefficients if a subsequent error when simplifying.
				B1	Condone absence of brackets only if followed by correct work eg $2x^2 = 4x^2$ must be soi for second B mark. $1 + x$ www
			$= 1 + x - ½\, x^2 + ½\, x^3 + \ldots$	B1 B1	… $- ½\, x^2$ www … $+ ½\, x^3$ www If there is an error in say the third coeff of the expansion, M0, B1, B0, B1 can be scored
			Valid for $\lvert x \rvert < 1/2$ or $-1/2 < x < 1/2$	B1	Independent of expansion $\lvert x\rvert \leq 1/2$ and $-1/2 \leq x \leq 1/2$ are actually correct in this case so we will accept them. Condone a combination of inequalities. Condone also, say $-1/2 < \lvert x\rvert < 1/2$ but not $x < 1/2$ or $-1 < 2x < 1$ or $-1/2 > x > 1/2$
				[5]	

Question			Answer	Marks	Guidance
3	(i)		$dV/dt = k\sqrt{V}$ $V = (½\,kt + c)^2$ $\Rightarrow$ $dV/dt = 2(½\,kt + c).½\,k$ $= k(½\,kt + c)$ $= k\sqrt{V}$	B1 M1 A1 A1 **[4]**	cao condone different k (allow MR B1 for $= kV^2$) $2(1/2\,kt + c) \times$ constant multiple of k (or from multiplying out oe; or implicit differentiation) cao www any equivalent form (including unsimplified) Allow SCB2 if $V=(1/2\,kt + c)^2$ fully obtained by integration including convincing change of constant if used Can score B1 M0 SCB2
	(ii)		$(½\,k + c)^2 = 10\,000 \Rightarrow ½\,k + c = 100$ $(k + c)^2 = 40\,000 \quad \Rightarrow \quad k + c = 200$ $\Rightarrow \quad ½\,k = 100$ $\Rightarrow \quad k = 200,\ c = 0$ $\Rightarrow\ V = (100t)^2 = 10000t^2$	B1 B1 M1 A1 **[4]**	substituting any one from $t = 1$, $V = 10{,}000$ or $t = 0$, $V = 0$ or $t = 2$, $V = 40{,}000$ into squared form or rooted form of equation (Allow $-/\pm100$ or $-/\pm200$) substituting any other from above Solving correct equations for both www (possible solutions are (200,0), (−200,0), (600, −400), (−600,400) (some from –ve root)) either form www SC B2 for $V = (100t)^2$ oe stated without justification SCB4 if justification eg showing substitution SC those working with $(k + c)^2 = 30{,}000$ can score a maximum of B1B0 M1A0 (leads to $k \approx 146$, $c \approx 26.8$)

Question			Answer	Marks	Guidance
4			LHS = $\sec^2\theta + \text{cosec}^2\theta$ $= \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}$ $= \dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}$ $= \dfrac{1}{\cos^2\theta\sin^2\theta}$ $=\sec^2\theta\ \text{cosec}^2\theta$ …………………………………………………………… **OR** $\sec^2\theta+\text{cosec}^2\theta=\tan^2\theta+1+\cot^2\theta+1=\sin^2\theta/\cos^2\theta+\cos^2\theta/\sin^2\theta+2$ $= \dfrac{\cos^4\theta + \sin^4\theta + 2\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$ $= \dfrac{(\cos^2\theta + \sin^2\theta)^2}{\sin^2\theta\cos^2\theta} = \dfrac{1}{\sin^2\theta\cos^2\theta} = \sec^2\theta\ \text{cosec}^2\theta$ …………………………………………………………… **OR working with both sides** **Eg** LHS $\sec^2\theta + \text{cosec}^2\theta = \tan^2\theta+1+\cot^2\theta+1=\tan^2\theta+\cot^2\theta+2$ RHS $=(1+\tan^2\theta)(1+\cot^2\theta)= 1+ \tan^2\theta+ \cot^2\theta+\tan^2\theta\cot^2\theta$ $=\tan^2\theta+\cot^2\theta+2=$LHS	M1 M1 A1 A1 M1 M1 A1 A1 M1 M1 A1 A1 **[4]**	**Use** of $\sec\theta = 1/\cos\theta$ and $\text{cosec}\theta = 1/\sin\theta$ not just stating adding use of $\cos^2\theta + \sin^2\theta = 1$ soi **AG** correct formulae oe adding use of Pythagoras **AG** Correct formulae used on one side Use of same formulae on other side Use of $\tan\theta\cot\theta = 1$ oe, dependent on both method marks Showing equal

Question			Answer	Marks	Guidance
5			$\sin(x + 45°) = \sin x \cos 45° + \cos x \sin 45°$ $= \sin x .1/\sqrt{2} + \cos x . 1/\sqrt{2}$ $= (1/\sqrt{2})(\sin x + \cos x) = 2\cos x$ $\Rightarrow \sin x + \cos x = 2\sqrt{2}\cos x$ *	M1 A1 A1	Use of **correct** compound angle formula **Since AG ,** $\sin x \cos 45° + \cos x \sin 45° = 2\cos x$ $\sin x + \cos x = 2\sqrt{2} \cos x$ only gets M1 need the second line or statement of $\cos 45° = \sin 45° = 1/\sqrt{2}$ oe as an intermediate step to get A1 A1
			$\Rightarrow \sin x = (2\sqrt{2} - 1) \cos x$ $\Rightarrow \tan x = 2\sqrt{2} - 1$ $\Rightarrow x = 61.32°$, $241.32°$	 M1 A1 A1 **[6]**	 terms collected and $\tan x = \sin x / \cos x$ used for first correct solution for second correct solution and no others in the range 2dp but allow overspecification ignore solutions outside the range SC A1 for both 61.3° and 241.3° SC A1 for both 1.07 and 4.21 radians (or better) SC A1 for incorrect answers that round to 61.3° and 180° + their ans eg 61.33° and 241.33° Do not award SC marks if there are extra solutions in the range.

Question			Answer	Marks	Guidance
6			$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y}{x(x+1)}$		
			$\Rightarrow \quad \int \frac{1}{y}\mathrm{d}y=\int \frac{1}{x(x+1)}\mathrm{d}x$	M1	correctly separating variables and intending to integrate (ie need to see attempt at integration or integral signs)
			$\frac{1}{x(x+1)}=\frac{A}{x}+\frac{B}{x+1}$ $\Rightarrow \quad 1 = A(x + 1) + B\,x$	M1	partial fractions soi
			$x = 0 \Rightarrow A = 1$	A1	$A = 1$ www
			$x = -1 \Rightarrow 1 = -B \Rightarrow B = -1$	A1	$B = -1$ www
			$\Rightarrow \quad \ln y=\int (\frac{1}{x}-\frac{1}{x+1})\mathrm{d}x=\ln x-\ln(x+1)+c$	A1	ft their A,B condone absence of c or ln$c$
			$x = 1, y = 1 \Rightarrow 0 = 0 -\ln 2 + c \Rightarrow c = \ln 2$	B1	evaluating their c **at any stage** dependent on x and y terms all being logs of correct form but **do not award following incorrect log rules,** ft their A,B**.** c could be say a decimal. (eg $y = x/(x + 1) + c$ then c being found is B0)
			$\Rightarrow \quad \ln y = \ln x - \ln(x + 1) + \ln 2 = \ln (2x/(x + 1))$	M1	correctly combining lns and antilogging throughout (must have included the constant term). Apply this strictly. Do not allow if c is included as an afterthought unless completely convinced. ft A,B Logs must be of correct form ie not following say $\int \frac{1}{x(x+1)}\mathrm{d}x=\ln(x^2+x)$ unless ft from partial fractions and $B = 1$
			$\Rightarrow \quad y = 2x/(x + 1)$	A1	cao www $\quad (y=e^{.693}\left(\frac{x}{x+1}\right)$ loses final A1) **NB** evaluating c and log work can be in either order. eg $y = cx/(x + 1)$, at $x = 1$, $y = 1$, $c = 2$
				[8]	

Question			Answer	Marks	Guidance
7	(i)		$\theta = -\pi/2$: O (0, 0) $\theta = 0$: P (2, 0) $\theta = \pi/2$: O (0, 0)	B1 B1 B1 **[3]**	Origin or O, condone omission of (0, 0) or O Or, say at P $x = 2$, $y = 0$, need P stated Origin or O, condone omission of (0,0) or O
7	(ii)		$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{2\cos 2\theta}{-2\sin\theta} = -\frac{\cos 2\theta}{\sin\theta}$ When $\theta = \pi/2$ $dy/dx = -\cos \pi/ \sin\pi/2 = 1$ When $\theta = -\pi/2$ $dy/dx = -\cos(-\pi)/ \sin(-\pi/2) = -1$ **Either** $1\times -1 = -1$ so perpendicular **Or** gradient tangent $=1 \Rightarrow$ meets axis at 45º , similarly, gradient $= -1 \Rightarrow$ meets axis at 45º oe	M1 A1 M1 A1 A1 **[5]**	their $dy/d\theta$ /$dx/d\theta$ any equivalent form www (not from $-2\cos 2\theta/2\sin\theta$) subst $\theta = \pi/2$ in their equation Obtaining $dy/dx = 1$,and $dy/dx = -1$ **shown** (or explaining using symmetry of curve) **www** justification that tangents are perpendicular **www** dependent on previous A1
7	(iii)		At Q, $\sin 2\theta = 1 \Rightarrow 2\theta = \pi/2$, $\theta = \pi/4$ $\Rightarrow$ coordinates of Q are $(2\cos \pi/4, \sin \pi/2)$ $= (\sqrt{2}, 1)$	M1 A1 A1 **[3]**	or, using the derivative, $\cos 2\theta = 0$ soi or their $dy/dx = 0$ to find θ. If the only error is in the sign or the coeff of the derivative in (ii), allow full marks in this part (condone $\theta = 45°$) www (exact only) accept $2/\sqrt{2}$
7	(iv)		$\sin^2\theta = (1 - \cos^2\theta) = 1 - \frac{1}{4}x^2$ $\Rightarrow$ $y = \sin 2\theta = 2\sin\theta\cos\theta$ $= (\pm)\, x\sqrt{(1 - \frac{1}{4}x^2)}$ $\Rightarrow$ $y^2 = x^2(1 - \frac{1}{4}x^2)$*	B1 M1 A1 A1 **[4]**	oe, eg may be $x^2 =$….. **Use** of $\sin 2\theta = 2\sin\theta\cos\theta$ subst for x **or** $y^2 = 4\sin^2\theta\cos^2\theta$ (squaring) either order oe squaring **or** subst for x either order oe **AG**

Question			Answer	Marks	Guidance
7	(v)		$V=\int_0^2 \pi x^2(1-\frac{1}{4}x^2)\,\mathrm{d}x$	M1	integral including correct limits but ft their '2' from (i) (limits may appear later) condone omission of d*x* if intention clear
			$=\int_0^2 (\pi x^2-\frac{1}{4}\pi x^4)\,\mathrm{d}x$		
			$=\pi\left[\frac{1}{3}x^3-\frac{1}{20}x^5\right]_0^2$	B1	$\left[\frac{1}{3}x^3-\frac{1}{20}x^5\right]$ ie allow if no π and/or incorrect/no limits (or equivalent by parts)
			$=\pi\left[\frac{8}{3}-\frac{32}{20}\right]$	A1	substituting limits into correct expression (including π) ft their '2'
			$= 16\pi/15$	A1 **[4]**	cao oe, 3.35 or better (any multiple of π must round to 3.35 or better)
8	(i)		$\overrightarrow{AA'}=\begin{pmatrix}2\\4\\1\end{pmatrix}-\begin{pmatrix}1\\2\\4\end{pmatrix}=\begin{pmatrix}1\\2\\-3\end{pmatrix}$	B1	finding $\overrightarrow{AA'}$ *or* $\overrightarrow{A'A}$ by subtraction, **subtraction must be seen** B0 if $\overrightarrow{AA'}$, $\overrightarrow{A'A}$ confused Assume they have found $\overrightarrow{AA'}$ if no label
			This vector is normal to $x + 2y - 3z = 0$	B1	reference to normal or ***n***, or perpendicular to $x + 2y - 3z = 0$, or statement that vector matches coefficients of plane and is therefore perpendicular, or showing AA' is perpendicular to **two** vectors in the plane
			M is (1½, 3, 2½) $x + 2y - 3z = 1½ + 6 - 7½ = 0$	M1	for finding M correctly (can be implied by two correct coordinates)
			$\Rightarrow$ M lies in plane	A1 **[4]**	showing numerical subst of M in plane = 0

Question			Answer	Marks	Guidance
8	(ii)		$\mathbf{r} = \begin{pmatrix}1\\2\\4\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\2\end{pmatrix} = \begin{pmatrix}1+\lambda\\2-\lambda\\4+2\lambda\end{pmatrix}$ meets plane when		
			$1 + \lambda + 2(2 - \lambda) - 3(4 + 2\lambda) = 0$	M1	subst of **AB** in the plane
			$\Rightarrow$ $-7 - 7\lambda = 0,\ \lambda = -1$	A1	
			So B is (0, 3, 2)	A1	cao
			$\overrightarrow{\mathrm{A'B}} = \begin{pmatrix}0\\3\\2\end{pmatrix} - \begin{pmatrix}2\\4\\1\end{pmatrix} = \begin{pmatrix}-2\\-1\\1\end{pmatrix}$	M1	or $\overrightarrow{BA'}$, ft **only** on their B (condone $\overrightarrow{A'B}$ used as $\overrightarrow{BA'}$ or no label) (can be implied by two correct coordinates)
			Eqn of line A'B is $\mathbf{r} = \begin{pmatrix}2\\4\\1\end{pmatrix} + \lambda\begin{pmatrix}-2\\-1\\1\end{pmatrix}$	B1 ft	$\begin{pmatrix}2\\4\\1\end{pmatrix}$ or their B +……
				A1 ft **[6]**	… λ × their $\overrightarrow{A'B}$ (or $\overrightarrow{BA'}$) ft only their B correctly
8	(iii)		Angle between $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}-2\\-1\\1\end{pmatrix}$	M1	correct vectors but ft their $\overrightarrow{A'B}$.Allow say, $\begin{pmatrix}-1\\1\\-2\end{pmatrix}$ and/or $\begin{pmatrix}2\\1\\-1\end{pmatrix}$ condone a minor slip if intention is clear
			$\Rightarrow$ $\cos\theta = \dfrac{1.(-2)+(-1).(-1)+2.1}{\sqrt{6}.\sqrt{6}}$	M1	correct formula (including cosθ) for their direction vectors from (ii) condone a minor slip if intention is clear
			= 1/6	A1	±1/6 or 99.6º from appropriate vectors only soi **Do not allow either A mark if the correct *B* was found fortuitously in (ii)**
			$\Rightarrow$ $\theta = 80.4°$	A1 **[4]**	cao or better

Question			Answer	Marks	Guidance
8	(iv)		Equation of BC is $\mathbf{r} = \begin{pmatrix}2\\4\\1\end{pmatrix} + \lambda\begin{pmatrix}-2\\-1\\1\end{pmatrix} = \begin{pmatrix}2-2\lambda\\4-\lambda\\1+\lambda\end{pmatrix}$		**NB this is not unique** eg $\begin{pmatrix}0\\3\\2\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$
			Crosses Oxz plane when $y = 0$	M1	For putting $y = 0$ in their line BC and solving for λ
			$\Rightarrow \lambda = 4$	A1	Do not allow either A mark if B was found fortuitously in (ii) for A marks need fully correct work only **NB this is not unique** eg $\begin{pmatrix}0\\3\\2\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$ leads to $\mu = -3$
			$\Rightarrow \mathbf{r} = \begin{pmatrix}-6\\0\\5\end{pmatrix}$ so $(-6, 0, 5)$	A1	cao
				[3]	

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Mathematics (MEI)

Advanced GCE

Unit **4754B:** Applications of Advanced Mathematics: Paper B

Mark Scheme for June 2012

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For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

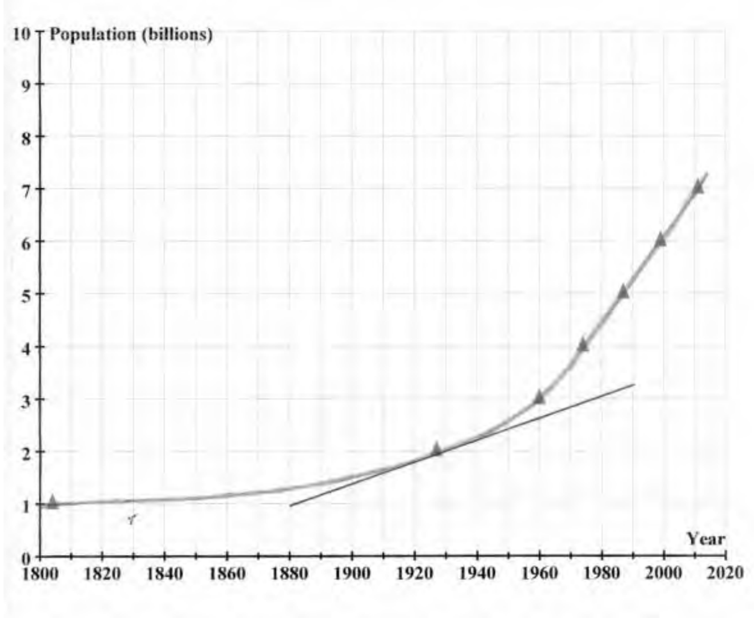
g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he / she wishes to be marked, then examiners should do as the candidate requests.
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1			Males 1.95 million, Females 2 million: Total 3.95 million	B1 **[1]**	accept 3.9-4 million allow 4000 thousand oe
2		(i)	 Gradient 0.018×10^9 giving 18 000 000 **(** people per year)	B1 M1 A1 **[3]**	curve and tangent drawn do not accept a polygon accept any reasonable **tangent** at the correct point (ie touches, not crosses) (NB B0M1A1 is possible if a full curve is not drawn) use of gradient (**<u>from tangent only</u>**) accept 12-28,000,000 do not accept unreasonable accuracy eg no more than 3sf (0.018 or 180 000, say, can score M1 A0) without tangent is M0 A0
		(ii)	$\frac{0.018\times10^9}{2\times10^9}\times100\% = 0.9\%$	M1 A1 **[2]**	allow follow through from previous part **for both marks** ie (**their (i)**/ 2×10^9) × 100%, for A mark do not allow more than 3sf could get M1A1 from say 0.018/2 × 100% without having scored A1 in (i)

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>3</td><td></td><td>(i)</td><td>$\frac{dp}{dt} = kp$
$\int \frac{dp}{p} = k\int dt$
$\ln p = kt + c$</td><td>
M1

A1</td><td>
separating variables correctly and intending to integrate

solving correctly, any form, need a constant</td></tr>
<tr><td></td><td></td><td></td><td>When $t = 0, p = p_0 \Rightarrow c = \ln p_0$
$\ln\left(\frac{p}{p_0}\right) = kt$
$p = p_0 e^{kt}$</td><td>A1

[3]</td><td>AG, fully correct derivation of given result including explicitly using initial condition (condone $t = 0$, $p = 10^9 = \boldsymbol{p_0}$)
SC1 for <u>verifying</u> the given result correctly ie differentiating p=… and substituting for p</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>$p_0 = 10^9$ so the 1927 figures give $2\times10^9 = 10^9 \times e^{k\times(1927-1804)}$
$\Rightarrow 123k = \ln 2$
$\Rightarrow k = 0.00563...$*</td><td>M1

A1

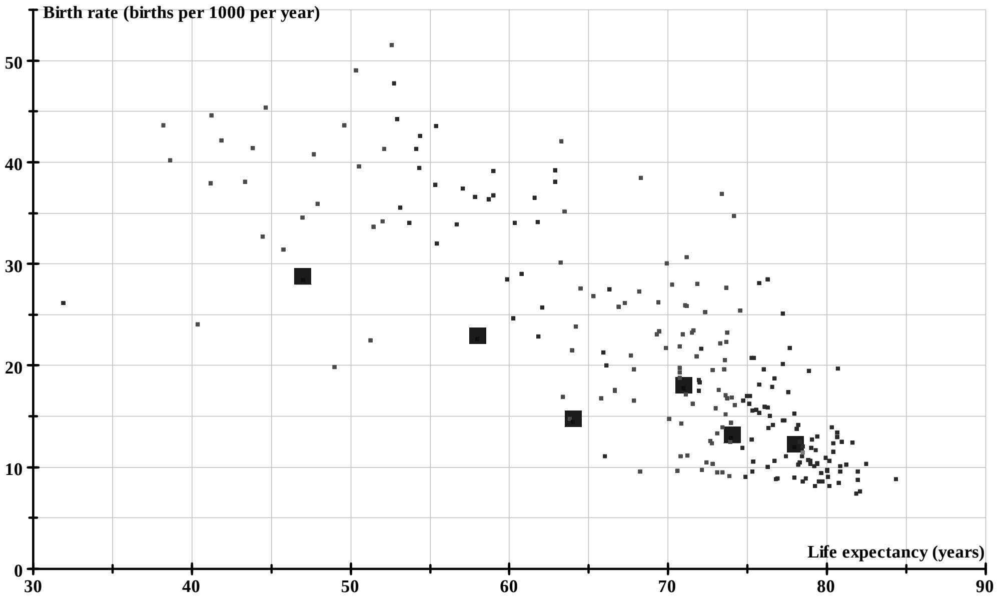
[2]</td><td>the equation must be correct (soi)
(10^9 could be cancelled)
cao AG so must SHOW enough, eg k = ln2/123 or 0.005635….</td></tr>
<tr><td>4</td><td></td><td></td><td>

Age group	**2010**	**2030**	**2050**	**2070**
80^+	1	**2**	**2.4**	**2.4**
60-79	10	**12**	**12**	**12**
40-59	20	**20**	**20**	**22**
20-39	20	**20**	**22**	**24.2**
0-19	20	**22**	**24.2**	**26.62**
Total	**71**	**76**	**80.6**	**87.22**

</td><td>B1
B1
B1

[3]</td><td>2030 column
2050 column
2070 column (need 2dp)

SCB2 for columns correct but no totals</td></tr>
</table>

Question			Answer	Marks	Guidance
5		(i)	The proportion of people in the 40-59 age group surviving into the 60-79 group is 80%.	B1 **[1]**	cao oe in fractions or decimals
		(ii)	The proportion of those in the 60-79 group surviving in to the 80^+ group is 40%.	B1 **[1]**	cao oe in fractions or decimals
6			The data show that, over the last 100 years, the birth rate in the UK has declined and the life expectancy has increased. This pattern is consistent with the UK having a developing economy.	B1 B1 **[2]**	1 mark for correct comments on changing (over time) birth rate and life expectancy. Not just negative correlation if no link (soi) to time 1 mark for linking to **development**
					This diagram does not need to be seen but some candidates may use it to help in their explanations.

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