GCE

Mathematics

Advanced GCE

Unit **4733:** Probability and Statistics 2

Mark Scheme for June 2012

Annotations

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (OCR) Statistics strand

a   Annotations should be used whenever appropriate during your marking.

        The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.
It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

        For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

        Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c   The following types of marks are available.

        M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

        A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

        B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g   Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.
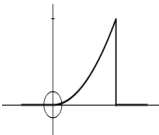
The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			Number CDs (sequentially) Select using random numbers **[2]**	B1 B1	List needn't be stated, but must mention CDs. *Not* "select numbers randomly". Hat, etc: B1B0 Systematic: 66 or 67 B1, random start B1	Assume sequential unless stated otherwise. If "number CDs randomly", B1 max unless "sort by number". Stratified: apply scheme
2	(i)		$\left(\dfrac{71.2 - 72.0}{\sigma / \sqrt{40}}\right) = -0.3853$ $[\sigma = 13.13,]\ \mathrm{Var}(V) = \mathbf{172.4}$ **[4]**	M1 A1 B1 A1	Standardise with Φ^{-1} & $\sqrt{40}$, allow cc, $\sqrt{}$ errors eg σ^2 Square roots and sign correct, no cc, no "1 –" error z in range ($\pm$) [0.385, 0.386] seen *Final* answer in range [172, 173], or 13.1^2 cwo	RHS must be Φ^{-1}, i.e. *not* 0.7411 or 0.2589 or 0.6368 or 0.35. "1 –" error or ×40/39: M1A0 [0.674 may be from "1 – 0.35 = 0.75"] Needs variance, not SD NB: Look out for $-13.1 \to 172$, M1A0B1A0
2	(ii)		Parent distribution not known n is large **[2]**	B1 B1	Or clear equivalent. Not "*sample* not normal" Or clear equiv, e.g. sample size > 30. Extras: max 1 "n large, $n > n_0$": B1 if $n_0 \geq 30$.	Don't bother about order of these statements. If numerical must be 30. Ignore "continuous".
3		α:	H$_0$: $p = \frac{1}{3}$ [or 0.33 or better] H$_1$: $p \neq \frac{1}{3}$ [or 0.33 or better] B(12, $\frac{1}{3}$) stated or implied P($\geq$ 7) = 1 – 0.9336 = 0.0664 > 0.025	B2 M1 A1 A1	Allow π, but $\mu = \frac{1}{3}$ etc B1. Any other letter, B0 One-tailed, or no symbol, B1 max B(12, $\frac{1}{3}$) stated or implied, allow for N(4,8/3), Po(4) Probability in range [0.066, 0.067] Explicit comparison with 0.025, or $2p$ with 0.05	Not $\mu = 4$ *(if in doubt, consult)* *If N used, or P($\leq$ 7) or P($=$ 7), no more marks* 1-tailed: A0 here regardless of value
		β:	CR is $\geq$ 8, 7 not in CR Probability is 0.0188	A1 A1	Needs explicit comparison of 7 with CV Must be $\geq$ 7, 0.019 or 0.0188 or better, allow 0.9812	Need to be clear that CR is being used – look for comparison with 7. See also ζ.
			Do not reject H$_0$. Insufficient evidence that statement is false. **[7]**	M1 A1√	Needs correct method, including like-with-like, correct tail, $\geq$ 7 (or $\leq$ 6). If CV, needs right tail A1 needs "evidence" or equivalent. "Statement" is enough context here	Allow from 1-tail. 0.9812 or 0.0188 or 0.0476: M0 unless "$\geq$ 7" stated or clearly using β. $\sqrt{}$ on their p/CR. Withhold if answer refers only to p.

Question			Answer	Marks	Guidance	
4	(i)		Crystals must occur independently of one another **[1]**	B1	Allow interpreted, or "randomly" but nothing else. Must be contextualised; no other answers included.	Ignore "singly" (meaningless in this context). But allow "probability… is independent"
4	(ii)		$e^{-3.2}\dfrac{3.2^5}{5!}$ = **0.114(0)** **[2]**	M1 A1	Formula, or .0608 or .1781 or .1075 or .1203 (tables) Answer a.r.t. 0.114, implies both marks	
4	(iii)		Po(2.368) $1 - e^{-2.368}(1 + 2.368 + \dfrac{2.368^2}{2})$ = **0.4219** **[3]**	M1 M1 A1	Po(0.74 × 3.2) stated or implied 1 – correct Poisson terms, their λ, allow ± 1 term Answer, a.r.t. 0.422, implies all 3 marks	Allow for 0.75 × 3.2 etc, e.g. Po(2.4) Don't allow second M1 from λ in tables, e.g. if MR, treat as E–1. If no working: don't give M1A0
4	(iv)		Po(32) ≈ N(32, 32) $1 - \Phi\left(\dfrac{35.5 - 32}{\sqrt{32}}\right)$ = 1 – Φ(.619) = **0.2681** **[5]**	M1 A1 M1 A1 A1	N(λ, λ) stated or implied, allow $\sqrt{\lambda}$ or λ^2 for var N(32, 32), allow $\sqrt{32}$ or 32^2 for var Standardise with λ and $\sqrt{\lambda}$ or λ, allow cc errors but not $\sqrt{n}$; both cc and $\sqrt{}$ correct Final answer, a.r.t. 0.268	Needs $\lambda \geq 15$ Can get (M0A0) M1A1 from $\lambda < 15$ Typically, no cc → 0.2203, or 32 → 0.4565, 3/5 (but needs evidence, not just answer)

Question			Answer	Marks	Guidance	
5			$H_0: \mu = 6.1$ $H_1: \mu \neq 6.1$	B2	Both: B2. One error, B1, but $\bar{x}$, x, r etc: 0. 6.2: B0	
			$\hat{\mu} = \bar{x} = 6.2$	B1	6.2 [31/5] seen somewhere (other than hypotheses)	
			$\hat{\sigma}^2 = \dfrac{80}{79}\left(\dfrac{3126}{80} - 6.2^2\right) = 0.643$	M1	Correct formula for biased estimate [0.635 or 127/200]	If single formula used, M2 or, if wrong, allow M1 for divisor 79 anywhere
				M1	Divide by 79 somewhere	
		α:		A1	Variance estimate, a.r.t. 0.643, can be implied	[254/395 leading to 127/15800]
			$z = \dfrac{6.2 - 6.1}{\sqrt{0.643/80}} = 1.115$	M1	Standardise their 6.2 with reasonable variance attempt, needs 80, allow cc	*80 needed, otherwise M0 and no more marks* *If clearly $\mu = 6.2$ used, no more marks*
			$[1 - \Phi(1.115) = 0.1325 > 0.05]$	A1	$z \in [1.11, 1.12]$ (not −) or $p \in [0.1323, 0.1333]$	A1 uses number used for comparison
			$1.115 < 1.645$	A1	Compare z with 1.645 (allow −1.645 if $z < 0$) or p (< 0.5) with 0.05	Withhold if inequality incorrect or if 1-tailed Must be consistent signs/tails and like-with-like
		β:	CV $\quad 6.1 + 1.645 \times \sqrt{\dfrac{0.643}{80}}$ $= 6.247$ and $6.2 < 6.247$	M1 A1 A1√	$6.1 + z\sqrt{(\sigma^2/80)}$, allow $\pm$, √ errors CV, a.r.t. 6.25, needs $z = 1.645$, allow biased $\hat{\sigma}^2$ Compare 6.2 with CV from + sign, √ on z (but not σ)	Allow $6.2 -$ (or $\pm$) but no more marks afterwards If no 79 earlier but used here, recovers M1A1 E.g. $1.96 \rightarrow 6.276$ or $1.282 \rightarrow 6.215$ [gets M1A0A1
			Do not reject H_0. Insufficient evidence that pH value is not 6.1	M1 A1√ **[11]**	Needs essentially correct method and comparison, needs 80 but no need for correct variance Needs context and "evidence" or equivalent, ft on their z/p/CV	First conclusion wrong: M0A0 even if second correct. "1.115 > 1.645 so do not reject H_0" etc: (A0)M1A1
Notes:			Biased estimate used : typically gets B2B1 M1M0A0 M1A0A1 M1A1 [total 8]		$\bar{x}$ and μ interchanged: allow final M1A1 if *anywhere* right, but if always wrong (in hypotheses and z) M0A0. This would typically get B0B0B1 M1M1A1 M1A0A0 M0A0 [total 5]	

Question			Answer		Marks	Guidance	
6	(i)		B(32, 0.4)		B1	B(32, 0.4) stated or implied, e.g. by Po(12.8)	Poisson [0.09888], or exact [0.046269]:B1max
			$\approx$ N(12.8, 7.68)		M1A1	N(their attempt at np, npq); N(12.8, 7.68)	SC: B(12.8, 7.68/32): M1A0
			Valid as 12.8 and 19.2 > 5		B1	Or "n large and p close to 0.5". Not npq or 7.68 > 5.	Allow np and nq both asserted > 5
			$1 - \Phi\left(\dfrac{17.5 - 12.8}{\sqrt{7.68}}\right)$		M1	Standardise, their np,npq, allow wrong/no cc or no $\sqrt{}$	$\div 32$: M0
					A1	17.5 and $\sqrt{npq}$ correct	
			$[= 1 - \Phi(1.696)]$ = **0.0449**		A1	Answer, a.r.t. 0.045	
					[7]		
6	(ii)		B(90, 0.01)		B1	B(90, 0.01) stated or implied.	Exact [0.049003]: B1 max.
			$\approx$ Po(0.9)		M1	Po(their attempt at np)	Don't treat $p = 0.1$ as MR. If $np > 5$, M0M0
			$e^{-0.9}\dfrac{0.9^3}{3!}$ = **0.0494**		M1	Correct formula or use of tables, e.g. 0.1646 or 0.0112	No working, wrong answer $\Rightarrow$ M0A0, but right answer $\Rightarrow$ M1A1 provided clearly Po
					A1	Final answer in range [0.049, 0.05) [i.e., *not* 0.05]	SC: B(90, 0.1), N(9, 8.1), [0.015, 0.016] cwo B2
					[4]		
7	(i)			Values of X close to a are more likely than those close to 0	M1	Positive parabola (only), through 0, nothing below x-axis	$k < 0$: M0 even if $k > 0$ as well.
					A1	Clear truncation at ends	Don't need any scales, vertical line at a etc. Can be vertical at A, needn't be horizontal at O.
					B1	Withhold if concept misunderstood. Need to have probability of *values* (not of *occurring*); not just shape. Allow for U-shape but nothing else	E.g.: "More likely to *occur* for x close to a": B0. Ignore extra comments like "exponential"
					[3]		
7	(ii)	(a)	$\displaystyle\int_0^a kx^2\,dx = 1 \Rightarrow k = \dfrac{3}{a^3}$		M1	Attempt to integrate kx^2, ignore limits	Must attempt integration
					A1	Correct limits and equate to 1	
			$\displaystyle\int_0^a \dfrac{3}{a^3}x^3\,dx = \dfrac{9}{2} \Rightarrow a = 6$		M1	Attempt to integrate kx^3, ignore limits	Must attempt integration
					A1	Correct limits and equate to 4.5	Don't need k in terms of a here
					A1	One correct equation connecting k and a, can be implied	$ka^3 = 3$ or $ka^4 = 18$, a.e. simplified form
					A1	Correctly obtain $a = 6$ only	No marks explicitly for k [= 1/72 or 0.01388...]
					[6]		

Question			Answer	Marks	Guidance
7	(ii)	(b)	$\int_0^6 \frac{1}{72} x^4 \mathrm{d}x \qquad [= \frac{108}{5}]$ $21.6 - 4.5^2 \ = \mathbf{1.35}$	M1 A1 A1 **[3]**	Attempt to integrate kx^4, their a, k, can be algebraic Subtract 4.5^2 (given in question) 1.35 or exact equivalent only	Must attempt integration; limits 0, $a\sqrt{}$ Somewhere [=27/20]
8	(i)		$30 + 1.645 \times \dfrac{8}{\sqrt{18}} = 33.102$ so CR is $\bar{X} > 33.1$	M1 A1 A1 A1√ **[4]**	$30 + z \times 8/\sqrt{18}$, allow $\sqrt{}$ errors, cc 1.645, requires + only 33.1 a.r.t. 33.10 $\geq$ their RH CV√, allow $\leq$ their LH CV *as well*, allow $>$, allow no letter or X but no other letter	Allow $\pm$ but not $-$ only. No 18: 0 in this part. Don't allow "accept if $\leq$ 33.1, reject if $>$ 33.1" Inequality required in final line
	(ii)		Type I [error]	B1 **[1]**	Nothing else unless it's just an amplification. Allow "Type 1"	
	(iii)		B(20, 0.05): P($\geq$ 4) = 0.0159 so unlikely that $\mu = 30$	M1 A1 A1√ **[3]**	B(20, 0.05) stated or implied. Not B(20, 1/5) Probability, a.r.t. 0.016 Justified conclusion, e.g. "I think $\mu = 30$ as not less than 0.01". FT on their p.	No reason: A0. Not over-assertive. But "I think $\mu = 30$ as probability is small" is A0.
	(iv)		$\dfrac{33.1 - \mu}{8/\sqrt{18}} = -0.253$ $\mu = \mathbf{33.58}$	M1 A1 A1 A1 **[4]**	Needs Φ^{-1}, their CV, SD right or same as in (i), allow cc Signs correct, can be implied by answer $>$ their CV z in range $(\pm)[0.25, 0.26]$ Final answer $33.55 \leq \mu \leq 33.60$, 4 SF needed.	*Not* 30. Allow omission of $\sqrt{18}$ only if omitted in (i). "1 $-$" errors: can get M1A0A1 Typically 32.62 probably gets 2/4.

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