

Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4754A
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

- 1 Solve the equation $\frac{4x}{x+1} - \frac{3}{2x+1} = 1$. [5]
- 2 Find the first four terms in the binomial expansion of $\sqrt{1+2x}$. State the set of values of x for which the expansion is valid. [5]
- 3 The total value of the sales made by a new company in the first t years of its existence is denoted by £ V . A model is proposed in which the rate of increase of V is proportional to the square root of V . The constant of proportionality is k .
- (i) Express the model as a differential equation.
- Verify by differentiation that $V = (\frac{1}{2}kt + c)^2$, where c is an arbitrary constant, satisfies this differential equation. [4]
- (ii) The value of the company's sales in its first year is £10 000, and the total value of the sales in the first two years is £40 000. Find V in terms of t . [4]
- 4 Prove that $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \operatorname{cosec}^2\theta$. [4]
- 5 Given the equation $\sin(x + 45^\circ) = 2 \cos x$, show that $\sin x + \cos x = 2\sqrt{2} \cos x$.
- Hence solve, correct to 2 decimal places, the equation for $0^\circ \leq x \leq 360^\circ$. [6]
- 6 Solve the differential equation $\frac{dy}{dx} = \frac{y}{x(x+1)}$, given that when $x=1$, $y=1$. Your answer should express y explicitly in terms of x . [8]

Section B (36 marks)

7 Fig. 7a shows the curve with the parametric equations

$$x = 2 \cos \theta, \quad y = \sin 2\theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}.$$

The curve meets the x -axis at O and P. Q and R are turning points on the curve. The scales on the axes are the same.

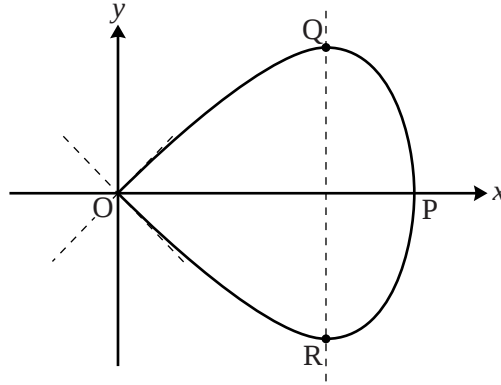


Fig. 7a

- (i) State, with their coordinates, the points on the curve for which $\theta = -\frac{\pi}{2}$, $\theta = 0$ and $\theta = \frac{\pi}{2}$. [3]
- (ii) Find $\frac{dy}{dx}$ in terms of θ . Hence find the gradient of the curve when $\theta = \frac{\pi}{2}$, and verify that the two tangents to the curve at the origin meet at right angles. [5]
- (iii) Find the exact coordinates of the turning point Q. [3]

When the curve is rotated about the x -axis, it forms a paperweight shape, as shown in Fig. 7b.

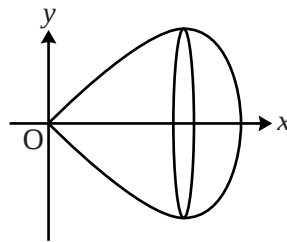


Fig. 7b

- (iv) Express $\sin^2 \theta$ in terms of x . Hence show that the cartesian equation of the curve is $y^2 = x^2(1 - \frac{1}{4}x^2)$. [4]
- (v) Find the volume of the paperweight shape. [4]

- 8 With respect to cartesian coordinates $Oxyz$, a laser beam ABC is fired from the point $A(1, 2, 4)$, and is reflected at point B off the plane with equation $x + 2y - 3z = 0$, as shown in Fig. 8. A' is the point $(2, 4, 1)$, and M is the midpoint of AA' .

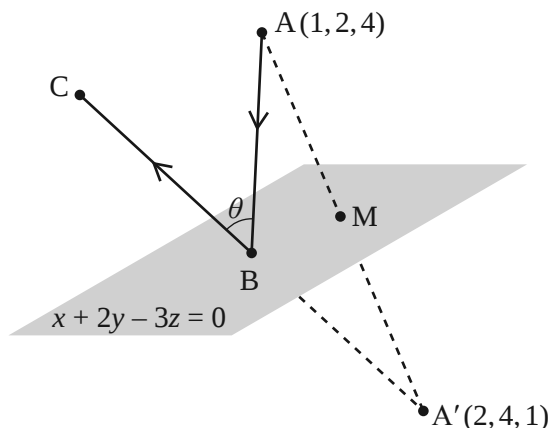


Fig. 8

- (i) Show that AA' is perpendicular to the plane $x + 2y - 3z = 0$, and that M lies in the plane. [4]

The vector equation of the line AB is $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

- (ii) Find the coordinates of B , and a vector equation of the line $A'B$. [6]
- (iii) Given that $A'BC$ is a straight line, find the angle θ . [4]
- (iv) Find the coordinates of the point where BC crosses the Oxz plane (the plane containing the x - and z -axes). [3]

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Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754B Applications of Advanced Mathematics (C4) Paper B: Comprehension

Candidates answer on the Question Paper.

OCR supplied materials:

- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator
- Rough paper

Duration: Up to 1 hour



Candidate forename		Candidate surname	
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Centre number						Candidate number				
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INSTRUCTIONS TO CANDIDATES

- The Insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- The Insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You may find it helpful to make notes and to do some calculations as you read the passage.
- You are **not** required to hand in these notes with your Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **8** pages. Any blank pages are indicated.

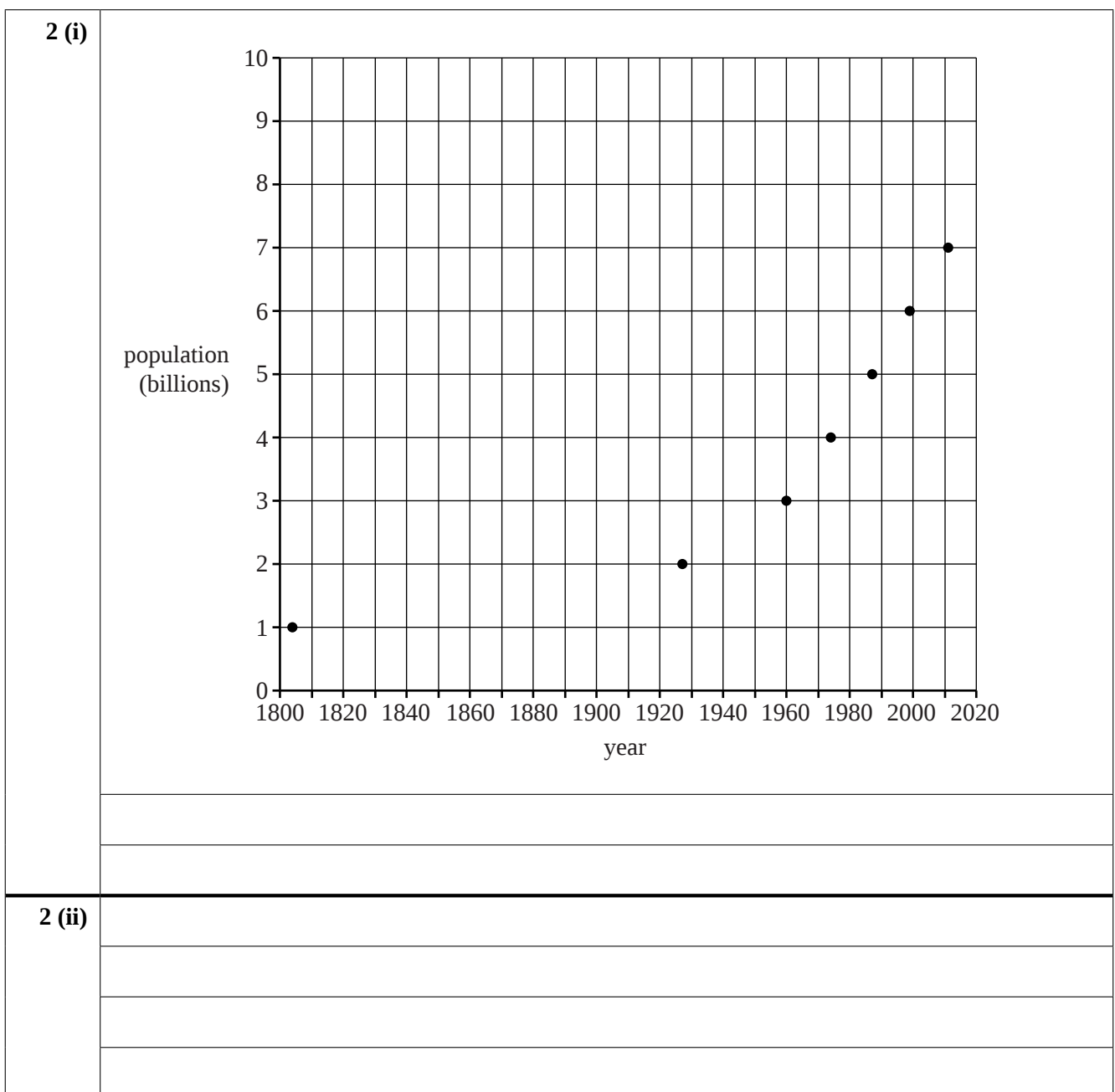
- 1 Use Fig. 4 to estimate the number of 50–54 year olds in the UK in 2001. (These were born in the post World War 2 baby boom.) [1]

1	

- 2 A copy of Fig. 2 is given below.

- (i) Join the points with a curve and hence estimate the rate of population growth in the year 1927 in people per year. [3]

- (ii) Estimate this rate as a percentage of the population at that time. [2]



- 3 (i) In line 21, the solution of the differential equation $\frac{dp}{dt} = kp$ is stated to be $p = p_0 e^{kt}$.

Use integration to derive this result.

[3]

- (ii) The article then goes on to say

“If a model is to be valuable in this context, it must be possible to use it to predict the size of the world population in the future. So, as a test case, the first two data points in Table 1 should allow the later values to be predicted. These data points are

$$1804 \quad t = 0, p = p_0 = 10^9,$$

$$1927 \quad t = 123, p = 2 \times 10^9,$$

and these correspond to $k = 0.00563 \dots$.”

Show how this value of k is obtained.

[2]

3 (i)	
3 (ii)	

- 4 In Table 6, the population profile of an imaginary country was predicted. Complete the table subject to the same general assumptions except that, after 2010:

- the average number of children per female is 2.2;
- 60% of those in the 40–59 age group survive into the 60–79 age group;
- 20% of those in the 60–79 age group survive into the 80+ age group.

[3]

4

Age group	2010	2030	2050	2070
80+	1			
60–79	10			
40–59	20			
20–39	20			
0–19	20			
Total	71			

As in Table 6, the figures are in millions.

- 5 In constructing Table 6, some assumptions were made about the proportion of people surviving from one age group to the next. Use Table 6 to find

(i) the proportion of people in the 40–59 age group surviving into the 60–79 age group, [1]

(ii) the proportion of those in the 60–79 age group surviving into the 80+ age group. [1]

5 (i)	
5 (ii)	

6 This table refers to the UK. It gives life expectancy and birth rate every 20 years from 1901 to 2001.

Year	Life expectancy	Birth rate (births/1000)
1901	47	28.5
1921	58	22.7
1941	64	14.5
1961	71	17.8
1981	74	12.9
2001	78	12.0

Explain how these data relate to the conclusions of the article.

[2]

[A copy of Fig. 7 is given below. You do not need to use it but may find it helpful.]

6

